Speed, time and average speed

1 One relation, three readings

$$D = S \times T$$

Same road	Speed and time in inverse ratio
Same time	Distance follows speed
Minutes	Into hours first: 10 min $= \frac{1}{6}$ h, 15 min $= \frac{1}{4}$ h
km/h to m/s	$\times \frac{5}{18}$, so 72 km/h $= 20$ m/s

Speed $\frac{5}{6}$ of usual, 12 min late

Time becomes $\frac{6}{5}$ of usual: $\frac{1}{5}$ extra

$\frac{1}{5} \times T = 12$

$T = 60$ **min**

Trap: The new time: 72 min.

2 Late plus early

The gap between two arrivals is late + early.

36 km/h: 10 min late; 45 km/h: 6 min early

Times $5 : 4$; gap $10 + 6 = 16$ min is 1 part

Time at 36 km/h $= 80$ min

$D = 36 \times \frac{80}{60} = 48$ **km**

Trap: Using $10 - 6$: 12 km.

3 Only after the breakdown

Before the change both days match. The whole delay sits on the rest.

After $\frac{2}{3}$ of a trip, $60 \to 45$ km/h; 20 min late

$\frac{d}{45} - \frac{d}{60} = \frac{1}{3} \Rightarrow d = 60$ km

That is the last third

Trip $= 180$ **km**

Trap: Delay on the whole trip: 60 km.

4 Equal distances: harmonic mean

Average speed is total distance over total time.

$$\bar{v} = \frac{n}{\frac{1}{v_1} + \cdots + \frac{1}{v_n}}$$

Equal stretches at $20, 30, 60$ km/h

$\frac{3}{\frac{1}{20} + \frac{1}{30} + \frac{1}{60}} = \frac{3}{\frac{6}{60}}$

$= 30$ **km/h**

Trap: The plain average, 36.7 km/h.

5 Equal time or equal distance?

Equal times: plain average. Equal distances: $\frac{2ab}{a+b}$, always lower.

240 km at 40 and 60 km/h

Half the time: 50 km/h, 4.8 h

Half the way: $\frac{2 \cdot 40 \cdot 60}{100} = 48$ km/h, 5 h

Half the way is 12 **min slower**

Trap: Saying they tie.

Relative speed and trains

1 Make one body stand still

Facing each other	Add the speeds
Same direction	Subtract the speeds
Then	Time = gap ÷ relative speed

Thief at 40 km/h; jeep 15 min later at 60 km/h

Head start $40 \times \frac{1}{4} = 10$ km, closed at 20 km/h: $\frac{1}{2}$ h

Caught $60 \times \frac{1}{2} = 30$ km from the post

Trap: The head start, 10 km.

2 After the crossing: $t^2 = t_1 t_2$

$$t = \sqrt{t_1 t_2}, \qquad \frac{v_A}{v_B} = \sqrt{\frac{t_B}{t_A}}$$

A needs 16 min more, B 25 min

They crossed at $\sqrt{16 \times 25} = 20$ min

$v_A : v_B = \sqrt{25} : \sqrt{16} = 5 : 4$

Trap: No square root: $25 : 16$.

3 The shuttling fly

Don't add the legs. Time the meeting, then multiply.

60 km apart at 12 and 18 km/h; fly 40 km/h

They meet in $\frac{60}{30} = 2$ h

Fly: $40 \times 2 = 80$ km

Trap: The gap, 60 km.

4 Buses at fixed intervals

Gap between buses = bus speed × interval. You close it at the relative speed.

Every 12 min at 30 km/h: gap 6 km

Walking towards: $6 = (30 + v) \times \frac{10}{60}$

$v = 36 - 30 = 6$ **km/h**

Trap: $30 \times \frac{12 - 10}{12} = 5$ km/h.

5 Pole and platform

A pole costs the train its own length. A platform costs both lengths.

Pole 10 s; 300 m platform 25 s

Extra 15 s is the platform: 20 m/s

Length $= 20 \times 10 = 200$ m

Trap: $20 \times 25 = 500$ m, train + platform.

6 Head-on and same way

Lengths add. Head-on gives the sum of speeds, same way the difference.

$150 + 210 = 360$ m; 12 s and 60 s

Sum 30 m/s, difference 6 m/s

Faster $= 18$ m/s

Trap: The sum, 30 m/s.

7 Two walkers, two signs

Walker going the same way: subtract. Coming towards: add.

Cyclist 5 m/s: 20 s; walker towards at 1 m/s: 12 s

$20(v - 5) = 12(v + 1) \Rightarrow v = 14$

Length $= 20 \times 9 = 180$ m

Trap: $v - 1$ for both: 120 m.

Boats, escalators, averages

1 Down, up and still water

$$\text{Down} = B + W, \quad \text{Up} = B - W, \quad B = \frac{\text{Down} + \text{Up}}{2}, \quad W = \frac{\text{Down} - \text{Up}}{2}$$

30 km down and back in 8 h; stream 2 km/h

$\frac{30}{B+2} + \frac{30}{B-2} = 8 \Rightarrow 2B^2 - 15B - 8 = 0$

$(B-8)(2B+1) = 0$; check: $3 + 5 = 8$ h

$B = 8$ **km/h**

Trap: $\frac{60}{8} = 7.5$ km/h is the average speed, not B.

2 Round trip is always slower

$$\bar{v} = \frac{B^2 - W^2}{B}$$

$B = 10$ km/h, $W = 2$ km/h

$\frac{100 - 4}{10}$

$= 9.6$ **km/h**

Trap: The still-water speed, 10 km/h.

3 Speed for a target average

Two equal halves: $\frac{2av}{a+v}$ hits the target. It must stay under $2a$.

First half 30 km/h; target 40 km/h

$\frac{60v}{30+v} = 40 \Rightarrow 20v = 1200$

$v = 60$ **km/h**

Trap: $2 \times 40 - 30 = 50$ km/h.

4 The floating ball

Ride with the water: the ball stands still. Out and back take the same time.

Ball drops at a bridge; turn after 30 min; caught 6 km down

30 min out + 30 min back: the ball drifted 1 h

Stream = 6 km/h

Trap: 6 km in 30 min: 12 km/h.

5 Count the stairs

Steps on the stair $n = (m \pm e)\,t$: her steps plus the escalator's.

30 steps in 15 s; 40 steps in 10 s

$30 + 15e = 40 + 10e \Rightarrow e = 2$ steps/s

$n = 30 + 15 \times 2 = 60$

Trap: Her own 30 steps.

6 Stopped escalator

Add rates, not times.

Walking down it 20 s; standing 60 s

$\frac{1}{w} = \frac{1}{20} - \frac{1}{60} = \frac{1}{30}$

$w = 30$ **s**

Trap: $60 - 20 = 40$ s.

To and fro, and circular tracks

1 To and fro: $(2n-1)L$

From opposite ends, meeting n comes when the two have covered $(2n-1)L$ together. From the same end it is $2nL$. Keep the faster under twice the slower, or it overtakes and adds meetings.

300 m pool, opposite ends, 2 and 3 m/s; 4th meeting

$7 \times 300 = 2100$ m at 5 m/s

420 **s**

Trap: $4L$: 240 s.

2 First meeting on a loop

Opposite	$\dfrac{L}{a+b}$
Same way	$\dfrac{L}{a-b}$

400 m, 5 and 3 m/s

Opposite: $\dfrac{400}{8} = 50$ s

Same way: $\dfrac{400}{2} = 200$ **s**

Trap: Adding speeds when they run the same way.

3 Back at the start: LCM

LCM of the lap times. For fractions: LCM of the tops over HCF of the bottoms.

400 m, 5 and 3 m/s, same way

Laps 80 s and $\dfrac{400}{3}$ s

$\dfrac{\text{LCM}(80, 400)}{\text{HCF}(1, 3)} = 400$ **s**

Trap: The first meeting anywhere: 200 s.

4 Distinct meeting points

Reduce the speeds to $m : n$. Opposite: $m+n$ points. Same way: $m-n$.

10 and 6 m/s, opposite

$10 : 6 = 5 : 3$

$5 + 3 = 8$ **points**

Trap: Not reducing: 16.

5 The k-more-rounds equation

Same way for T: the faster gains k laps, $kL = (a-b)\,T$. Opposite: $L = (a+b)\,t$ at the first meeting.

400 m, 5 and 3 m/s

In 800 s: $(5-3) \times 800 = 1600$ m

$= 4$ **laps more**

Trap: Using $a+b$: 16 laps.

6 Before you answer

Minutes into hours? · Equal distances: harmonic mean, never the plain average. · Facing: add; chasing: subtract. · Did the train's own length go in? · Asked the boat's speed or the average?