Assume the work

1 Work = Rate × Time

Given times? Take the work as the LCM of the times, so every rate is whole.

Rate	Work ÷ own time
Together	Add the rates
Against (leak, drain)	Subtract the rate

A 12 days, B 18 days

Work = LCM = 36; rates 3 and 2

Together = $\frac{36}{5} = 7.2$ **days**

Trap: Averaging the times: 15 days.

2 The missing worker

Total rate minus the rates you know.

A 10, B 15, all three 5 days

Work 30: $3 + 2 + C = 6$

C $= 1 \Rightarrow 30$ **days**

Trap: Forgetting B: 10 days.

3 Taking turns

Run whole cycles, then check who works the last part day.

A 8, B 12, alternate days, A first

Work 24; two days $= 3 + 2 = 5$

8 days $\rightarrow 20$; day 9 (A) $\rightarrow 23$

Day 10 (B) needs 1 of 2: 9.5 **days**

Trap: $\frac{24}{5} \times 2 = 9.6$ days.

4 Pair sums, then halve

Adding A+B, B+C, C+A counts everyone twice.

A+B 12, B+C 15, C+A 20 days

Work 60: $5 + 4 + 3 = 12$

A+B+C $= 6 \Rightarrow 10$ **days**

Trap: Not halving: 5 days.

5 Pay by work done

Share = rate × days that person worked.

A 20, B 30. A alone 4 days, then A, B, C 6 days. ₹9,000

Work 60: A does 12 alone; 48 left in 6 days $\rightarrow 8$ a day

C $= 8 - 3 - 2 = 3$ a day $\rightarrow 18$ work

C gets $\frac{18}{60} \times 9{,}000 =$ **₹2,700**

Trap: Splitting by rate, $\frac{3}{8}$: ₹3,375.

Assume the rate

1 Efficiency is a rate

Twice as efficient	$A = 2B$
80% **as efficient**	$A = \frac{4}{5}B$, so $5A = 4B$
Thrice the time	Rate is a third: $3A = B$
Time and rate	Inverse: time ratio flips

2 Percent more efficient

Turn the % into a rate ratio, then flip it for time.

A is 25% more efficient than B; B 20 days

Rates $5 : 4 \Rightarrow$ times $4 : 5$

A $= 16$ days

Trap: Cutting the time by 25%: 15 days.

3 More time, less rate

Times $3 : 2$ means rates $2 : 3$.

A takes 50% more time than B; together 12 days

Rates $2 : 3$; work $= 5 \times 12 = 60$

A alone $= \frac{60}{2} = 30$ days

Trap: $12 \times 1.5 = 18$ days.

4 Each takes extra time

A alone takes a more than A+B together, B alone b more.

$$t^2 = ab$$

$a = 9$ h, $b = 16$ h

$t = \sqrt{144} = 12$ h

A $= 21$ h, B $= 28$ h

Trap: Answering 16: that is only the extra time.

5 Three people, one time ratio

Clear the fractions with an LCM, as in Ratio.

$T_A = 2T_B = 3T_C$; together 6 days

Rates $1 : 2 : 3$; work $= 6 \times 6 = 36$

A alone $= 36$ days

Trap: Using the times as rates: 12 days.

Crews and man-hours

1 Two crews, one job

Work = (crew rate) × days. Equal jobs, so equate.

4 men +6 women: 8 days; 3 men +7 women: 10 days

$(4m + 6w)8 = (3m + 7w)10$

$2m = 22w \Rightarrow m = 11w$

Work = $(44 + 6) \times 8 = 400$

10 women: 40 days

Trap: Treating a man as a woman: 8 days.

2 Man-hours

$$M_1 D_1 H_1 = \text{work}_1, \quad M_2 D_2 H_2 = \text{work}_2$$

$18 \times 8 \times 10 = 1{,}440$ is 40%

Job = 3,600; left 2,160

24 workers, 6 h: $\frac{2160}{144} = 15$ **days**

Trap: Reusing the whole job: 10 days.

3 Extra people for a deadline

Find the man-days left, then subtract the crew you have.

12 men, 10 days: $\frac{1}{3}$ of a wall

120 man-days = $\frac{1}{3} \Rightarrow 360$

Left 240 in 10 days: 24 men

12 more men

Trap: Answering 24, the whole crew.

4 Three kinds of worker

Equate two crews to remove one kind, then another pair. Divide the two results to get the last relation. Then find the work from any crew.

5 Efficiency boost midway

Take each worker as 4, so +25% is a whole 5.

40 workers, 12 km in 60 days; 2 km after 20

$40 \times 4 \times 20 = 3{,}200$ per 2 km $\Rightarrow 19{,}200$

Left 16,000 in 40 days = 400 a day

Half boosted: $20 \times 5 + 20 \times 4 = 180$

$180 + 4n = 400 \Rightarrow n = 55$ **more**

Trap: Ignoring the boost: 60.

6 Someone leaves

Work done together, then the rest by one.

A+B 12 days; A leaves after 4

Work 60: 20 done; B does 40 in 20 days

B = 2, A = 3: A alone 20 days

Trap: Adding the days: 24.

Pipes and patterns

1 A leak is negative work

Fill rates add, drain rates subtract.

Fills in 6 h; with a leak 8 h

Work 24: 4 in, net 3

Leak 1 an hour: 24 h

Trap: Subtracting times: 2 h.

2 Count the pipes

Same tank, so equate the two set-ups.

5 fill $+2$ drain: 6 h; 3 fill $+2$ drain: 12 h

$(5f - 2d)6 = (3f - 2d)12 \Rightarrow f = 2d$

Work $= 8 \times 6 = 48$

2 fill $+1$ drain $= 3$: 16 h

Trap: Taking $f = d$: 18 h.

3 Rosters

One full cycle, then day by day. Every 1st, 2nd, 3rd day: cycle 6.

A 10, B 15, C 30; A+B, B+C, C+A, repeat

Work 30: cycle $5 + 3 + 4 = 12$

6 days $\rightarrow 24$; day $7 \rightarrow 29$

Day 8 needs 1 of 3: $7\frac{1}{3}$ days

Trap: $\frac{30}{12} \times 3 = 7.5$ days.

4 One more worker every day

Person-days: $1 + 2 + 3 + \ldots$

One worker: 45 days

$1 + 2 + \cdots + n = 45$

$n = 9$ days

5 Cancel the shared stretch

Same start on two days: the matching part did the same work. Compare only what differs.

Same 9 am start. Ravi alone: done 6 pm

Ravi till 3 pm, then Sita alone: done 5 pm

Ravi 3 h $=$ Sita 2 h $\Rightarrow R : S = 2 : 3$

Work $= 9 \times 2 = 18$; together $\frac{18}{5} = 3$ h 36 min

$12{:}36$ pm

Trap: Flipping the ratio: $2{:}24$ pm.

6 Pipes come in whole numbers

One equation, two unknowns? Counts are whole and bounded.

Taps 4 h, each leak 20 h; 1 to 7 leaks; full in 1 h

Work 20: $5n - k = 20$

k must be a multiple of 5: $k = 5$

$n = 5$ taps

Trap: Ignoring the leaks: 4 taps.

7 One person's share

Count only that person's days, part day included.

A 10, B 15, C 30; A+B, A+C, B+C, repeat

Work 30: cycle $5 + 4 + 3 = 12$; 6 days $\rightarrow 24$

Day 7 (A+B) $\rightarrow 29$; day 8 (A+C) $\frac{1}{4}$ day

A: $3 \times 5\frac{1}{4} = 15.75$ of $30 = \frac{21}{40}$

Trap: A's share of the rates: $\frac{1}{2}$.

8 Before you answer

Work $=$ LCM of the times. · More time means less rate. · Pay follows work done, not rate. · Drains are negative. · Same start? Cancel the shared part. · Last day: who is working?