# Ratio



## 1 The core idea

$a : b : c$ means the real values are $ak,\ bk,\ ck$. Find $k$ and you have everything.

$$\text{Share of A} = \frac{a}{a+b+c} \times \text{Total}$$

## 2 Fractions to a ratio

Multiply every term by the LCM of the denominators.

$\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$

$\times 12$ on every term

$= 6 : 8 : 9$

**Trap:** Reading it as $1 : 2 : 3$ (the numerators).

## 3 Split a total

One part = total ÷ sum of the parts.

₹1,350 in $2 : 3 : 4$

$1 \text{ part} = \frac{1350}{9} = 150$

Largest − smallest $= 2$ parts

$= 2 \times 150 =$ ₹300

**Trap:** Dividing by 3, the number of people.

## 4 Merge on a common term

Make the shared term equal using its LCM.

$A : B = 2 : 3, \quad B : C = 4 : 5$

$\text{LCM}(3, 4) = 12$

$A : B = 8 : 12, \quad B : C = 12 : 15$

$A : B : C = 8 : 12 : 15$

**Trap:** Joining them directly as $2 : 3 : 5$.

## 5 Ratio chain, A to D

Multiply the fractions. Every middle term cancels.

$A : B = 2 : 3,\ B : C = 4 : 5,\ C : D = 6 : 7$

$\frac{A}{D} = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{48}{105}$

$A : D = 16 : 35$

**Trap:** Adding the ratios.

## 6 Equal value: pA = qB = rC

More pieces means each is cheaper, so the ratio **flips**.

$3S = 4V = 6C, \quad \text{LCM} = 12$

$S : V : C = \frac{12}{3} : \frac{12}{4} : \frac{12}{6}$

$S : V : C = 4 : 3 : 2$

**Trap:** Writing $3 : 4 : 6$.

# Ratio, continued



## 7 Fractions of the rest

Assume the total = LCM of the denominators. Not 100.

$\frac{1}{3}$ rent, $\frac{2}{5}$ of the rest food, saves ₹$24{,}000$

Take total $= \text{LCM}(3, 5) = 15$

Rent $5$, rest $10$, food $4$, saves $6$

$6$ units $=$ ₹$24{,}000 \Rightarrow 1$ unit $=$ ₹$4{,}000$

**Total** $= 15 \times 4{,}000 =$ ₹$60{,}000$

**Trap:** Taking $\frac{2}{5}$ of the whole salary (₹$90{,}000$).

## 8 Ratio split inside a chain

Put the ratio's **sum** into the LCM too.

$\frac{1}{3}$ in hostel, rest $1 : 2 : 4$, bike – bus $= 60$

Total $= \text{LCM}(3, 7) = 21$

Hostel $7$, rest $14 \Rightarrow 2, 4, 8$

$8 - 2 = 6$ units $= 60 \Rightarrow 1$ unit $= 10$

**Total** $= 21 \times 10 = 210$

**Trap:** Applying $1 : 2 : 4$ to the whole total $(140)$.

## 9 "A multiple of" means minimum k

Write the quantity as (number) × $k$, then let $k$ supply the missing factors.

Pens : pencils $= 4 : 9$

Gap $= 5k$, a multiple of $15$ and $6$

$\text{LCM}(15, 6) = 30 \Rightarrow k = 6$

**Total** $= 13k = 78$

**Trap:** Stopping at $k = 3$: $15$ divides it, $6$ does not.

## 10 Person in the middle

Count the person: total = parts + 1.

Ahead : behind $= 3 : 5$, total under $100$

$8k + 1 < 100 \Rightarrow k \leq 12$

**Most ahead** $= 3 \times 12 = 36$

**Trap:** Forgetting the $+1$.

## 11 Unknown integer in a fraction

Solve in terms of $k$, then keep only whole-number cases.

Gives $\frac{1}{3}$ and $\frac{k}{5}$ of total $x$, keeps $40$

$$x\left(1 - \frac{1}{3} - \frac{k}{5}\right) = 40 \Rightarrow x = \frac{600}{10 - 3k}$$

$k = 1$ fails, $k = 2 \Rightarrow 150$, $k = 3 \Rightarrow 600$

**Sum of totals** $= 750$

## 12 Same amount added to both

The **difference stays fixed**. Match it across both ratios.

$A : B = 3 : 5$, both $+6$ gives $9 : 13$

Gap: $2k = 4$ new parts, so $1$ new part $= \frac{k}{2}$

$9$ new parts: $4.5k = 3k + 6 \Rightarrow k = 4$

**Values** $12$ **and** $20$

# Partnership



## 1 The one rule

$$\text{Profit share} \propto \text{Capital} \times \text{Time}$$

Picture each partner as a rectangle: height = money, width = months. Profit follows the area. Same time? Split by capital. Same capital? Split by time. A loss is shared the same way.

## 2 Joining late

Months counted = 12 − months late.

Arjun ₹40k for 12 months

Bela ₹60k, joins after 4 months

$40 \times 12 = 480, \quad 60 \times 8 = 480$

Equal, so ₹27,000 splits evenly

**₹13,500 each**

**Trap:** Splitting by capital only $(2:3)$.

## 3 Money changes midway

Add the area of each stretch.

₹10k for 4 months, then ₹20k for 8

$10 \times 4 + 20 \times 8 = 40 + 160$

$= 200$ **(thousand-months)**

## 4 Working partner's cut

Salary or manager's share comes off **first**.

Profit ₹50,000, Kavya takes 20% first

$20\% = ₹10{,}000$, rest ₹40,000

Rest in $3:2 \Rightarrow$ ₹24,000, ₹16,000

**Kavya** $= 10{,}000 + 24{,}000 = ₹34{,}000$

**Trap:** Splitting ₹50,000 first.

## 5 Reverse partnership

Profit ratio known, capital missing. Work back from the areas.

A: ₹30k $\times 12 = 360$. B joins after 4

Profit $3:2 \Rightarrow$ B's area $= 240$

**B's capital** $= \frac{240}{8} = ₹30{,}000$

**Trap:** Using 12 months for B.

## 6 Time unknown

Set area ratio = profit ratio and solve for months.

P: ₹20k $\times 12 = 240$. Q: ₹30k $\times t$

Profit $2:1 \Rightarrow \frac{240}{30t} = \frac{2}{1}$

$t = 4$ **months, so Q joined after** 8

## 7 Check options by gaps

Money gaps give ratio gaps. Skip the equations.

P is 4k above Q, Q is 3k above R

Gaps must be $4:3$

$13:10:8$ has gaps $3:2$, no

$14:10:7$ **has gaps** $4:3$**, yes**

# Variation



## 1 The forms

| | | |
|---|---|---|
| **Direct** | $y \propto x$ | $\frac{y_2}{y_1} = \frac{x_2}{x_1}$ |
| **Inverse** | $y \propto \frac{1}{x}$ | $x_1 y_1 = x_2 y_2$ |
| **Square** | $y \propto x^2$ | $\frac{y_2}{y_1} = \left(\frac{x_2}{x_1}\right)^2$ |
| **Square root** | $y \propto \sqrt{x}$ | $\frac{y_2}{y_1} = \sqrt{\frac{x_2}{x_1}}$ |
| **Joint** | $y \propto \frac{xz}{w}$ | $y = k \cdot \frac{xz}{w}$ |

You rarely need $k$. Compare the two situations as a ratio and it cancels.

## 2 Inverse

The product stays constant.

$40$ km/h takes $6$ hours

$40 \times 6 = 240$ km

**At $60$ km/h: $\frac{240}{60} = 4$ hours**

**Trap:** Scaling up: $6 \times \frac{60}{40} = 9$.

## 3 Square

New value = old × (size ratio)².

Price $\propto$ diameter$^2$. $8$ inch costs ₹$200$

$12$ inch: $200 \times \left(\frac{12}{8}\right)^2 = 200 \times \frac{9}{4}$

$= ₹450$

**Trap:** Linear thinking (₹$300$).

## 4 Square root

New value = old × √(size ratio).

$y \propto \sqrt{x}$. At $x = 16,\ y = 12$

At $x = 25$: $12 \times \sqrt{\frac{25}{16}} = 12 \times \frac{5}{4}$

$y = 15$

**Trap:** Forgetting the root: $12 \times \frac{25}{16} = 18.75$.

## 5 Broken gem

Value $\propto$ weight². Break it in $a : b$:

$$\text{Loss} = \frac{2ab}{(a+b)^2} \times \text{Value}$$

₹$50{,}000$ breaks $2 : 3$

$\frac{2 \times 2 \times 3}{5^2} = \frac{12}{25}$

**Loss = ₹$24{,}000$**

**Trap:** Thinking the pieces still add up to ₹$50{,}000$.

## 6 Rate versus total

Find the rate, then multiply by time.

Fuel per hour $\propto$ speed². $40$ km/h: $2$ L/hr

At $60$: $2 \times \left(\frac{60}{40}\right)^2 = 4.5$ L/hr

$300$ km takes $5$ hours

**Fuel $= 4.5 \times 5 = 22.5$ L**

**Trap:** Answering $4.5$: that's per hour.

## 7 Partly fixed, partly varying

$y = \text{fixed} + \text{rate} \times x$. Two data points give both.

$20$ students ₹$5{,}000$; $30$ students ₹$7{,}000$

Rate $= \frac{2000}{10} = 200$, fixed $= 1{,}000$

**$50$ students = ₹$11{,}000$**

**Trap:** Scaling directly (₹$12{,}500$).