The price chain

1 Every step is a factor

Draw a box for each price. Put a factor on each arrow. Multiply, never add.

Profit $p\%$	$\times\left(1+\frac{p}{100}\right)$: 20% is $\times\frac{6}{5}$
Loss or discount $d\%$	$\times\left(1-\frac{d}{100}\right)$: 12.5% off is $\times\frac{7}{8}$
Profit $\%$	Always on CP, unless the question names another base

Maker $\xrightarrow{+20\%}$ dealer $\xrightarrow{+25\%}$ shop $\xrightarrow{-20\%}$ buyer pays ₹1,800

$\frac{6}{5}\times\frac{5}{4}\times\frac{4}{5}=\frac{6}{5}$

Maker's cost $=1800\times\frac{5}{6}=₹1{,}500$

Trap: Adding $20+25-20=25\%$: ₹1,440.

2 Profit on SP, not CP

Here $\frac{1}{5}$ of SP is $\frac{1}{4}$ of CP.

Charges ₹1,500; profit is 20% of that

Profit $=\frac{1}{5}\times 1500=300$

CP $=1500-300=₹1{,}200$

Trap: $1500\div 1.2=₹1{,}250$.

3 Gain counted in items

Selling n gains the SP of k: CP of n = SP of $n-k$.

12 pens gain the SP of 3

CP of 12 = SP of 9

Profit $=\frac{3}{9}=33\frac{1}{3}\%$

Trap: $\frac{3}{12}=25\%$.

4 The middleman's cut

Run the chain on 100, then scale to the rupee gap.

P $\xrightarrow{-20\%}$ Q $\xrightarrow{+20\%}$ R; R pays ₹48 less than P's cost

$100\to 80\to 96$: gap $4=₹48$, so $100=₹1{,}200$

Q's profit $=16\times 12=₹192$

Trap: 20% of P's cost: ₹240.

5 Same price, up and down

Equal SPs at $x\%$ profit and $x\%$ loss always lose $\frac{x^2}{100}\%$.

Two phones at ₹9,600: $+20\%$ and -20%

CPs 8000 and 12000; SP 19200

Loss $=₹800\ (4\%)$

Trap: "No profit, no loss".

Parts of a lot

1 Take ₹1 a unit and count rupees

Fractions of a lot sell at different rates? Price each unit at ₹1 (or ₹10). Lost units still cost money but bring none.

60 bulbs: a third at $+20\%$, a sixth break; target $+10\%$

Need 66. Sold: $20 \times 1.2 = 24$. Broken: 10. Left: 30

30 bulbs must bring $66 - 24 = 42$

$\frac{42}{30} = 1.4 \Rightarrow 40\%$ **profit on the rest**

Trap: Spreading 42 over 40 bulbs: 5%.

2 Alligation on profit %

A loss is a negative profit. Flip the gaps.

₹12,000: TV $+25\%$, fridge -5%, overall $+15\%$

Gaps $25 - 15 = 10, 15 - (-5) = 20$

TV : fridge $= 20 : 10 = 2 : 1$

Fridge $=$ ₹4,000

Trap: Not flipping: ₹8,000.

3 The rest covers the shortfall

Every rupee the first part falls short, the rest must make up.

6 kg at ₹300 + 4 kg at ₹550: cost ₹4,000

Target $\times 1.2 =$ ₹4,800

8 kg at ₹420 bring ₹3,360

Last 2 kg: $\frac{1440}{2} =$ ₹720 **a kg**

Trap: $1.2\times$ the average cost: ₹480.

4 The fixed cost

Two sales levels: the change in result over the change in units is the margin.

50 cakes: loss ₹500; 80 cakes: profit ₹4,000

Margin $= \frac{4000 + 500}{30} =$ ₹150

Rent $= 50 \times 150 + 500 =$ ₹8,000

Trap: Dropping the loss sign: ₹7,000.

5 Both prices change

Write both SPs in terms of the old CP.

10% cheaper, ₹30 cheaper: 25%, not 20%

$1.2c - 30 = 1.25 \times 0.9c = 1.125c$

$0.075c = 30 \Rightarrow c =$ ₹400

Trap: The 25% on the old cost: ₹600.

Markup and discount

1 CP, MP, SP: one more box

Markup is on CP. Discount is on MP. Profit is on CP.

$$\text{SP} = \text{CP} \times \left(1 + \frac{m}{100}\right) \times \left(1 - \frac{d_1}{100}\right) \times \left(1 - \frac{d_2}{100}\right)$$

Marked 50% up, then 20% and 10% off

$\frac{3}{2} \times \frac{4}{5} \times \frac{9}{10} = \frac{27}{25} = 1.08$

Profit $= 8\%$

Trap: $50 - 20 - 10 = 20\%$.

2 Profit on the marked price

Put everything on an MP of 100.

20% off; profit $= 20\%$ of MP

MP 100: SP 80, profit 20, CP 60

Markup $= \frac{40}{60} = 66\frac{2}{3}\%$

Trap: Profit on CP: $\frac{80}{1.2}$ gives 50%.

3 Free items are a discount

Buy n get 1 free: the factor is $\frac{n}{n+1}$.

Buy 4 get 1, plus 10% off; profit 8%

$m \times \frac{9}{10} \times \frac{4}{5} = 1.08$

$m = 1.5 \Rightarrow$ **markup** 50%

Trap: Free item as 25% off ($\times\frac{3}{4}$): 60%.

4 Same shop, deeper discount

Find MP as a multiple of CP once. Then reuse it.

10% off gives $+17\%$

$0.9\,\text{MP} = 1.17\,\text{CP} \Rightarrow \text{MP} = 1.3\,\text{CP}$

25% **off:** $1.3 \times 0.75 = 0.975$**, a** 2.5% **loss**

Trap: $17 - 15 = 2\%$ profit.

5 Successive discounts

Multiply what is left, then subtract from 1.

10%, 20%, 25% off

$\frac{9}{10} \times \frac{4}{5} \times \frac{3}{4} = 0.54$

Net discount $= 46\%$

Trap: $10 + 20 + 25 = 55\%$.

Dishonest shopkeeper

1 The weight is a factor too

Factor = what he is paid for ÷ what he really gives.

Gives 900 g as 1 kg	$\times\frac{1000}{900}=\times\frac{10}{9}$
Gets 1200 g for 1 kg	$\times\frac{1200}{1000}=\times\frac{6}{5}$
Mixes x kg into W kg	$\times\frac{W+x}{W}$

20% up, 10% off, a 900 g "kilo"

$$\frac{6}{5}\times\frac{9}{10}\times\frac{10}{9}=\frac{6}{5}$$

Profit $= 20\%$

Trap: 900 g as $+10\%$: 1.08×1.1 gives 18.8%.

2 Cheats both ends

One factor for the supplier, one for the customer.

Gets 1200 g per kg, gives 800 g; sells at cost

$$\frac{6}{5}\times\frac{1000}{800}=\frac{6}{5}\times\frac{5}{4}=\frac{3}{2}$$

Profit $= 50\%$

Trap: 800 g as $\times1.2$: 44%.

3 Stones for a target

Price factor times weight factor equals the target.

Sells 12.5% below cost; adds stones to 30 kg dal; wants $+5\%$

$$\frac{7}{8}\times\frac{30+x}{30}=\frac{21}{20}$$

$$30+x=36$$

Stone $= 6$ **kg**

Trap: 17.5% of 30: 5.25 kg.

4 Work back to the weight

Cancel the price factors first. What is left comes from the weight.

25% up, 20% off, yet $+25\%$

$\frac{5}{4}\times\frac{4}{5}=1$, so $\frac{1000}{w}=\frac{5}{4}$

$w = 800$ **g**

Trap: 25% less weight: 750 g.

5 Before you answer

Profit on CP or on MP? · Multiply the factors, never add them. · Lost or free units cost money but bring none. · A false weight is $\frac{\text{paid for}}{\text{given}}$.