Percentage

1 Percent is a fraction

Write it as a fraction first. It keeps every later step whole.

50%	$\frac{1}{2}$	25%	$\frac{1}{4}$	12.5%	$\frac{1}{8}$	37.5%	$\frac{3}{8}$
62.5%	$\frac{5}{8}$	87.5%	$\frac{7}{8}$	33.33%	$\frac{1}{3}$	66.67%	$\frac{2}{3}$
16.67%	$\frac{1}{6}$	8.33%	$\frac{1}{12}$	20%	$\frac{1}{5}$	40%	$\frac{2}{5}$
14.28%	$\frac{1}{7}$	28.57%	$\frac{2}{7}$	11.11%	$\frac{1}{9}$	22.22%	$\frac{2}{9}$
9.09%	$\frac{1}{11}$	27.27%	$\frac{3}{11}$	7.14%	$\frac{1}{14}$	6.25%	$\frac{1}{16}$

2 Spot the family

Multiples of 9.09% have 11 below. Multiples of 11.11% have 9 below. Multiples of 14.28% have 7 below.

27.27% of 88

$= \frac{3}{11} \times 88$

$= 24$

Trap: Multiplying by 0.2727: slow, and not exact.

3 Percent of: pick the easy side

$a\%$ of b equals $b\%$ of a.

18% of 50

$= 50\%$ of 18

$= 9$

4 Add up by fractions

Several shares of one total: use each fraction.

168 seats: 37.5% VIP, 14.28% press

$\frac{3}{8} \times 168 = 63, \quad \frac{1}{7} \times 168 = 24$

Left $= 168 - 63 - 24 = 81$

5 Percent versus percentage points

Points subtract. Percent divides by the old value.

40% becomes 50%

Change $= 10$ points

Rise $= \frac{10}{40} = 25\%$

Trap: Calling this a 10% rise.

Rises, falls, flips

1 Choose the multiplier

Easy % (20%)	Decimal	$\times 1.2$
Clean fraction (37.5%)	$1\pm$ fraction	$\times \frac{11}{8}$
Number is given	Add the part	$40 + 8 = 48$

Falling? Use $1-$ fraction. 9.09% down is $\times \frac{10}{11}$.

2 Going up by a fraction

$+\frac{a}{b}$ means multiply by $\frac{b+a}{b}$.

₹880 rises 37.5%

$880 \times \frac{11}{8}$

$= ₹1{,}210$

3 Going backwards

Divide by the multiplier. Never subtract the same %.

After $+25\%$, price $= ₹160$

Multiplier $\frac{5}{4}$

$160 \times \frac{4}{5} = ₹128$

Trap: 25% off 160 gives ₹120. The base has changed.

4 More than, flipped

A is $\frac{1}{n}$ more than B, so B is $\frac{1}{n+1}$ less than A.

$$\text{Less} = \frac{\text{Gap}}{\text{Bigger}}$$

Riya earns 25% more than Aman

$\frac{1}{4}$ more becomes $\frac{1}{5}$ less

Aman is 20% below Riya

Trap: Saying 25% less.

5 Price up, use down

Spend stays the same, so the same flip applies.

Price up $14.28\% = \frac{1}{7}$

Use down by $\frac{1}{8}$

Cut use by 12.5%

6 Comparing through a third person

Give the middle one 100, then walk both ways.

Ravi 20% above Sonu, 25% below Tara

Sonu 100, Ravi 120

Tara $= 120 \times \frac{4}{3} = 160$

Tara is 60% above Sonu

Trap: Adding: $20 + 25 = 45\%$.

Chains and groups

1 Fractions of the rest

Assume the LCM of the denominators. Not 100.

37.5% to schools, 20% of rest to libraries, $1{,}700$ left

$\text{LCM}(8, 5) = 40$

Schools 15, rest 25, libraries 5, left 20

20 **units** $= 1{,}700 \Rightarrow$ **total** $= 3{,}400$

Trap: Adding $37.5\% + 20\%$ gives $4{,}000$.

2 Percent plus a ratio split

The ratio's sum goes in the LCM too.

80% spent: painting 40%, others $3 : 5$

Total $= 5 \times 5 \times 8 = 200$

Spent 160, painting 64, others $36, 60$

$64 - 60 = 4$ units $= 240$

Total $= 12{,}000$

3 Two groups: a weighted average

Different group sizes, so never average the two %.

Seniors : juniors $= 3 : 2$. Take 500

40% of $300 = 120$; 65% of $200 = 130$

$\frac{250}{500} = 50\%$

Trap: Simple average: 52.5%.

4 Share of people, share of money

Take 100 people and 100 money.

$$\text{Average} = \frac{\text{Money share}}{\text{People share}}$$

20% of staff get 50% of pay

$\frac{50}{20} : \frac{50}{80}$

$= 4 : 1$

Trap: Reading it as $5 : 2$.

5 Fill the missing link

Work inward from both known ends.

$160 \to -12.5\% \to ? \to +20\% \to 210$

Forward: $160 \times \frac{7}{8} = 140$

Back: $210 \times \frac{5}{6} = 175$

Middle rise $= \frac{35}{140} = 25\%$

6 Whole-number counts

Smallest total = LCM of the denominators.

$37.5\% = \frac{3}{8}, \quad 16\% = \frac{4}{25}$

$\text{LCM}(8, 25) = 200$

Trap: Assuming 100: 37.5 people isn't possible.

Successive change

1 Two changes in a row

$$\text{Net} = a + b + \frac{ab}{100}$$

Use signs: a fall is negative.

$+20\%$, then -10%

$20 - 10 + \dfrac{20 \times (-10)}{100}$

$= +8\%$

Trap: Adding: $+10\%$.

2 Same up, then down

$$\text{Net} = -\frac{x^2}{100}$$

Order doesn't matter. It is always a fall.

$+20\%$, then -20%

$-\dfrac{400}{100}$

$= -4\%$

Trap: Expecting no change.

3 Long chains: multiply fractions

Three or more steps? Skip the formula.

Falls $25\%,\ 33.33\%,\ 20\%$, ends at 240

$\dfrac{3}{4} \times \dfrac{2}{3} \times \dfrac{4}{5} = \dfrac{2}{5}$

Start $= 240 \times \dfrac{5}{2} = 600$

4 Finding the rate: test the options

An equation is harder than checking choices.

Second raise is twice the first, total 168%

$(1 + x)(1 + 2x) = 1.68$

Try $x = 20\%$**:** $1.2 \times 1.4 = 1.68$

Trap: Splitting 68 into three.

5 Overall growth from two groups

It's alligation on the growth rates.

$5{,}000 \to 5{,}540$, overall $+10.8\%$

A $+10\%$, B $+12\%$

Gaps $1.2 : 0.8 = 3 : 2$

Type A $= 3{,}000$

6 Total = per unit × units

Divide the multipliers. Never subtract the %.

Sales $+26\%$, per store $+20\%$

$\dfrac{1.26}{1.2} = 1.05$

Stores up 5%

Trap: Subtracting: 6%.

7 Pass mark from a gap

The gap in % equals the gap in marks.

30% fails by 30; 45% passes by 30

$15\% = 60 \Rightarrow$ maximum 400

Pass mark $= 120 + 30 = 150$

$= 37.5\%$

8 Before you answer

Check the base: percent of what? · Fraction first, LCM next. · Asked for percent or points? · Rise or fall? Check the direction.