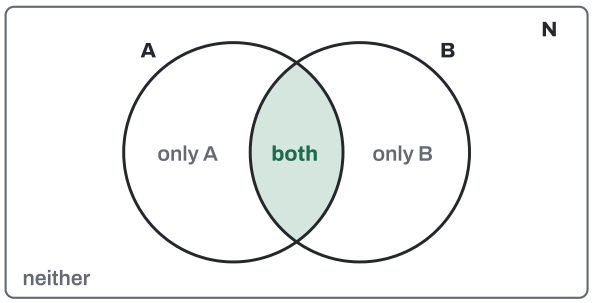
Two sets

1 Turn every phrase into regions

Two circles cut the box into four regions. "Likes A" is the whole circle, "only A" is its crescent, "both" is the lens. Check the none box first: if everyone is in at least one set, it is empty.

At least one	$A \cup B = A + B - \text{both}$
Only A	$A - \text{both}$
Exactly one	$A + B - 2 \times \text{both}$
Neither	$N - A \cup B$



2 Only means peel off the lens

Everyone in at least one: the total is the union.

$N = 120$, Hindi 75, English 62

Both $= 75 + 62 - 120 = 17$

Only English $= 62 - 17$

45 **students**

Trap: English read as only English: 62.

3 The none box

Take the neither people out of the box first.

$N = 150$, tea 85, coffee 70, neither 40

At least one $= 150 - 40 = 110$

Both $= 85 + 70 - 110$

45 **drink both**

Trap: $85 + 70 - 150$: 5.

4 Percent Venn, back to a count

Work in percent, scale at the end.

IPL 62%, World Cup 56%, neither 12%

Both $= 62 + 56 - 88 = 30\%$

30% is 450 people

Surveyed $= 450 \times \frac{100}{30} = 1500$

Trap: Neither ignored, both 18%: 2500.

5 Ratios sit on whole circles

"Chess players" includes the lens.

$N = 90$, chess $= 2\times$ carrom, both 15

$2k + k - 15 = 90$, so $k = 35$

Chess $= 70$

Only chess $= 70 - 15 = 55$

Trap: Ratio on the only regions: 50.

6 Exactly one: the lens goes twice

The union takes the lens off once, because the two circles count it twice. Exactly one wants it gone completely, so take it off twice.

$N = 240$, car 140, two-wheeler 110, neither 30

At least one $= 240 - 30 = 210$

Both $= 140 + 110 - 210 = 40$

Exactly one $= 140 + 110 - 2 \times 40 = 170$

Check by crescents: only car $= 140 - 40 = 100$, only two-wheeler $= 110 - 40 = 70$, and $100 + 70 = 170$.

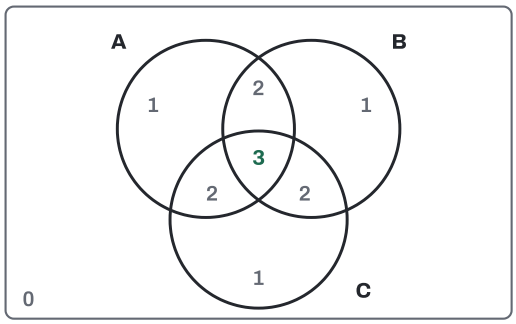
Trap: The union, the lens taken off once: 210.

Three sets

1 Count how often each person is counted

"Both A and B" always includes the centre. Adding the three circles counts an outer person once, a lens person twice, a centre person three times (the numbers in the figure). Every formula just fixes those counts. $\Sigma A = A + B + C$, ΣAB = the three pair counts.

At least one	$\Sigma A - \Sigma AB + ABC$
Exactly two	$\Sigma AB - 3\,ABC$
At least two	$\Sigma AB - 2\,ABC$
Exactly one	$\Sigma A - 2\,\Sigma AB + 3\,ABC$
Extra count	$\Sigma A - (\text{at least one}) = E_2 + 2E_3$



2 Fill from the centre out

Lens only = pair − centre.

Debate 60; with quiz 22, with dance 18; all three 12

Lens only: $22 - 12 = 10$ and $18 - 12 = 6$

Only debate $= 60 - 10 - 6 - 12$

32 students

Trap: $60 - 22 - 18$, centre never added back: 20.

3 Three circles and a none box

Union = N − none, then the formula.

$N = 200$, none 20; circles 110, 95, 70; pairs 45, 30, 35

Union $= 200 - 20 = 180$

$180 = 275 - 110 + ABC$

$ABC = 15$ **households**

Trap: 200 as the union: 35.

4 Exactly two: the centre goes three times

Each pair count holds the centre once.

Pairs 40, 31, 25; all three 18

$\Sigma AB = 96$

Exactly two $= 96 - 3 \times 18$

42 users

Trap: Centre taken off once: 78.

5 Exactly one

Singles − 2× pairs + 3× centre.

Circles 140, 120, 110; pairs 50, 40, 30; all three 20

$370 - 2 \times 120 + 3 \times 20$

$= 370 - 240 + 60$

190 shoppers

Trap: The at-least-one formula: 270.

6 The extra count

Two clubs: 1 extra count. Three: 2 extra.

120 students, all in a club; clubs 58, 64, 50

Extra $= 172 - 120 = 52$

$52 = 30 + 2E_3$ (exactly two = 30)

$E_3 = 11$ **in all three**

Trap: $52 - 30$, one extra each: 22.

7 Everyone in at least two

Two ticks each is the floor; the rest are third ticks.

40 members, each ≥ 2 hobbies; 30, 28, 26

Ticks = 84

Floor $= 2 \times 40 = 80$

All three $= 84 - 80 = 4$

Trap: $(84 - 40) \div 2$: 22.

Max and min

1 Slide the circles

A max/min question asks how far the circles can slide. Pushed together, the smaller one hides inside the bigger. Pulled apart, they stop when the box is full. With three circles, count ticks: N people can carry at most N outer ticks.

Both, two sets	$\max(0,\ A+B-N) \le \text{both} \le \min(A,B)$
All three, least	$ABC \ge \Sigma A - 2N$ (keep the non-likers apart)
All three, nobody outside	$E_2 + 2E_3 = \Sigma A - N$, so $E_3 \le \dfrac{\Sigma A - N}{2}$
Exactly one, two sets	$A + B - 2 \times \text{both}$: biggest when both is smallest

2 The overlap's range

Top: the smaller circle. Bottom: the box fills up.

$N = 80$, maths 52, science 45

Most both $= 45$

Least both $= 52 + 45 - 80 = 17$

Difference $= 45 - 17 = 28$

Trap: The larger circle as the most: 35.

3 Most for exactly one

Push the lens down to its floor.

$N = 90$, tea 60, coffee 50

Least both $= 60 + 50 - 90 = 20$

Exactly one $= 110 - 2 \times 20$

70 **people**

Trap: Both taken as 0: 110, more than 90.

4 Least for all three

Flip to the non-likers and keep them apart.

$N = 100$; cricket 80, football 75, hockey 70

Non-likers $20 + 25 + 30 = 75$

At most 75 miss a sport

At least $100 - 75 = 25$ **like all three**

Trap: Stopping after one pair, $80 + 75 - 100$: 55.

5 Most for all three, nobody outside

Each centre person uses two extra ticks.

$N = 60$, all like one; $40, 36, 32$

Extra ticks $= 108 - 60 = 48$

$E_2 + 2E_3 = 48$, so $E_3 \le 24$

24 **(outer regions** $16, 12, 8$**)**

Trap: The smallest circle: 32, which needs people outside.

Median, mode, range

1 Sort first, then count to the middle

Line the values up in order. The median is the middle one, the mode the most repeated value, the range the gap from the smallest to the largest. Only the mean feels every value.

Median	n odd: the $\frac{n+1}{2}$th value. n even: the mean of the $\frac{n}{2}$th and $\left(\frac{n}{2}+1\right)$th
Mode	The value seen most often, not its count
Range	Largest $-$ smallest
Add k to all	Mean, median, mode move by k; range stays
Change the largest	Mean moves by change $\div n$; median stays

2 Median from a table

Count matches, not rows.

Goals $0, 1, 2, 3, 4$ in $9, 8, 3, 4, 6$ matches

30 matches: the 15th and 16th

Running count 9, then 17

Both are 1: median 1 goal

Trap: The middle of the goals column: 2.

3 The median ignores the extremes

The largest moves, the middle doesn't.

7 numbers, mean 30, median 27

Largest $+14$: mean $+\frac{14}{7} = +2$

New mean 32, median still 27

$32 - 27 = 5$

Trap: Median moved by 2 too: 3.

4 Mean equals median, case by case

Where x sits decides the median.

$4, 6, 9, 13, x$; mean $= \frac{32+x}{5}$

$x \le 6$: $32 + x = 30$, $x = -2$

$6 \le x \le 9$: $32 + x = 5x$, $x = 8$

$x \ge 9$: $32 + x = 45$, $x = 13$

Sum $= -2 + 8 + 13 = 19$

Trap: Only the middle case: 8.

5 Mode, median and mean pin the list

The mode's copies must sit below the median.

Five positive integers, sum $5 \times 11 = 55$

$8, 8, 10, d, e$ with $d + e = 29$

$d = 10$ ties with the 8s, so $d \ge 11$

Largest $= 29 - 11 = 18$

Trap: $d = 10$: 19.

6 The smallest range

Lift the low values to just under the median, then split what is left as evenly as the rules allow.

Five different whole numbers, mean 20, median 18

Total 100; lows at most 16 and 17

Top two share $100 - 16 - 17 - 18 = 49$: $24, 25$

Range $= 25 - 16 = 9$

Trap: "Different" ignored, $18, 18, 18, 23, 23$: 5.