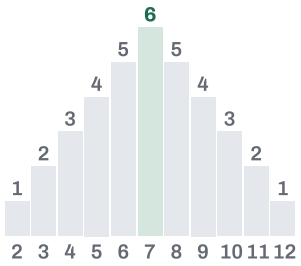
Count the sample space

1 Favourable over total

List equally likely outcomes: two dice are 36 ordered pairs, three coins 8 strings.

Probability	$\dfrac{\text{favourable}}{\text{total}}$
Or	$n(A) + n(B) - n(A \text{ and } B)$
Neither	$1 - P(A \text{ or } B)$
Two dice	Sum s has $6 - \lvert s - 7 \rvert$ ways out of 36
Coins	2^k outcomes; r heads in ${}^kC_r$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12



2 Or: take the overlap away

Both at once means a multiple of the LCM.

From 1 to 100: divisible by 4 or by 6

By 4: 25; by 6: 16

By both, so by 12: 8

$$P = \frac{25 + 16 - 8}{100} = \frac{33}{100}$$

Trap: The overlap counted twice: $\dfrac{41}{100}$.

3 Neither: one minus the union

Find "either" first, then take it away.

Neither a heart nor a face card

Hearts 13, faces 12, face hearts 3

Either $= 13 + 12 - 3 = 22$

$$P = \frac{52 - 22}{52} = \frac{15}{26}$$

Trap: $52 - 13 - 12 = 27$, face hearts taken twice: $\dfrac{27}{52}$.

4 Coins: count the orders

Head counts are not equally likely; strings are.

Three coins, exactly two heads

All strings: $2^3 = 8$

HHT, HTH, THH: ${}^3C_2 = 3$

$$P = \frac{3}{8}$$

Trap: $0, 1, 2, 3$ heads as four equal cases: $\dfrac{1}{4}$.

5 "A 3 and a 5" is two outcomes

Two different numbers can land either way round.

Two dice: one 3, one 5

$(3, 5)$ and $(5, 3)$

$$P = \frac{2}{36} = \frac{1}{18}$$

Trap: One order only: $\dfrac{1}{36}$.

6 Dice sums: $6 - \lvert s - 7 \rvert$

Seven has the most ways; each step away loses one.

Two dice, the sum is prime

Sums $2, 3, 5, 7, 11$

Ways $1 + 2 + 4 + 6 + 2 = 15$

$$P = \frac{15}{36} = \frac{5}{12}$$

Trap: Eleven sums as equal cases: $\dfrac{5}{11}$.

At least one: the opposite case

1 At least one $= 1-$ none

"At least one" has many cases and one opposite: none at all. Adding chances counts double successes twice.

At least one	$1 - q^n$, where $q = 1 - p$ is one failure
At most $n - 1$	$1-$ all n
At least one of a group	$1-$ none of that group
Fewest tries	Smallest n with $q^n \leq 1-$ target

2 At least one six

Every die must miss for "none".

Three dice, at least one six

No six on one die: $\frac{5}{6}$

No six on all: $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$

$P = 1 - \frac{125}{216} = \frac{91}{216}$

Trap: $3 \times \frac{1}{6}$: $\frac{1}{2}$.

3 Product even

The product is odd only when both are odd.

Two dice, the product is even

Both odd: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P = 1 - \frac{1}{4} = \frac{3}{4}$

Trap: Both taken as needing to be even: $\frac{1}{4}$.

4 The committee with at least one

Take away the committees with none.

3 from 4 men and 5 women, at least one man

All: ${}^9C_3 = 84$

No man: ${}^5C_3 = 10$

$P = \frac{74}{84} = \frac{37}{42}$

Trap: "One man, then any two": $\frac{4 \times {}^8C_2}{84} = \frac{112}{84}$, more than 1.

5 Fewest tries

Shrink the chance of missing every time.

Gets through with $\frac{1}{3}$; target at least $\frac{4}{5}$

Miss every try: $\left(\frac{2}{3}\right)^n \leq \frac{1}{5}$

$n = 3$: $\frac{8}{27}$, still above $\frac{1}{5}$

$n = 4$: $\frac{16}{81}$, below $\frac{1}{5}$

4 tries

Trap: Adding $\frac{1}{3}$ per try: 3 tries.

6 Solved by at least one

Independent people: unsolved only if both fail.

Chances $\frac{1}{3}$ and $\frac{1}{4}$

Both fail: $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

$P = 1 - \frac{1}{2} = \frac{1}{2}$

Trap: $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$.

Without replacement

1 The bag shrinks

"Together" and "one after another, not put back" are the same experiment. Only putting back, dice and coins keep the chances fixed.

Together	$\frac{{}^{a}C_{r} \times {}^{b}C_{n-r}}{{}^{a+b}C_{n}}$
One after another	$\frac{a}{N} \times \frac{b}{N-1}$, times the number of orders
Put back	${}^{n}C_{r}\, p^{r}\, q^{n-r}$
Unseen first draw	The second is like the first: $\frac{a}{N}$

2 Exactly two, taken together

Pick from each group, then multiply.

4 from 6 batters and 4 bowlers, exactly 2 bowlers

All teams: ${}^{10}C_{4} = 210$

${}^{4}C_{2} \times {}^{6}C_{2} = 6 \times 15 = 90$

$P = \frac{90}{210} = \frac{3}{7}$

Trap: As if put back, ${}^{4}C_{2} \left(\frac{2}{5}\right)^{2} \left(\frac{3}{5}\right)^{2} : \frac{216}{625}$.

3 One after another: count the orders

Each order is its own branch.

2 from 5 boys and 3 girls, exactly one girl

Boy, girl: $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

Girl, boy: $\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

$P = \frac{30}{56} = \frac{15}{28}$

Trap: One order only: $\frac{15}{56}$.

4 The second ball, unseen

Not looking at the first changes nothing.

5 red, 7 blue; the second is red

First red: $\frac{5}{12} \times \frac{4}{11} = \frac{20}{132}$

First blue: $\frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$

$P = \frac{55}{132} = \frac{5}{12}$

Trap: The first assumed red: $\frac{4}{11}$.

5 Parity of a sum

Even means same parity: both even or both odd.

Two different numbers from 1 to 10, sum even

All pairs: ${}^{10}C_{2} = 45$

Both even ${}^{5}C_{2} = 10$; both odd ${}^{5}C_{2} = 10$

$P = \frac{20}{45} = \frac{4}{9}$

Trap: Half the sums are even: $\frac{1}{2}$.

6 Put back: two sixes in four rolls

Count where the sixes go.

Places for two sixes: ${}^{4}C_{2} = 6$

Each: $\left(\frac{1}{6}\right)^{2} \left(\frac{5}{6}\right)^{2} = \frac{25}{1296}$

$P = 6 \times \frac{25}{1296} = \frac{25}{216}$

Trap: No orders: $\frac{25}{1296}$.

Independent events, given that

1 Multiply, and shrink

Independent chances multiply. "Given that" throws away every outcome that does not fit, then counts again.

Both	$P(A) \times P(B)$
Exactly one	$p_A\, q_B + q_A\, p_B$
Turns, first wins	$\dfrac{p}{1-q^2}$ for the one who starts
Given B	$\dfrac{n(A \text{ and } B)}{n(B)}$

2 Exactly one hits

Two cases: hit-miss and miss-hit.

Chances $\frac{2}{5}$ and $\frac{3}{4}$

$$\frac{2}{5} \times \frac{1}{4} + \frac{3}{5} \times \frac{3}{4}$$

$$P = \frac{2}{20} + \frac{9}{20} = \frac{11}{20}$$

Trap: Hit-miss only: $\frac{1}{10}$.

3 Taking turns

Win now, or both miss and start again.

First six wins; Kabir starts

$$P = \frac{1}{6} + \frac{25}{36}P$$

$$P = \frac{6}{36} \div \frac{11}{36} = \frac{6}{11}$$

Trap: As if turns were fair: $\frac{1}{2}$.

4 Given the sum

Only the cells of that sum are left.

Sum 8: $(2,6), (3,5), (4,4), (5,3), (6,2)$

A 3: $(3,5), (5,3)$

$$P = \frac{2}{5}$$

Trap: Out of all 36: $\frac{1}{18}$.

5 Two children

Strike out BB; three cases stay.

At least one girl: BG, GB, GG

Both girls: GG

$$P = \frac{1}{3}$$

Trap: The other child alone: $\frac{1}{2}$.

6 Which bag?

Red from A, out of all the red.

Red from A: $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

Red from B: $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$

$$P = \frac{3}{10} \div \frac{4}{10} = \frac{3}{4}$$

Trap: Red from bag A: $\frac{3}{5}$.