Counting and numbers

1 Count with slots

Draw one blank per decision. Fill the fussiest blank first (the units digit, the first digit, the named person), then the rest.

AND	Multiply: dal and sabzi and bread is $4 \times 3 \times \ldots$
OR	Add: roti or rice is $5 + 2$ choices for one slot
Zero	Can't lead. If 0 also matters at the end, split "ends in 0" from the rest
At least one	All − none
Sum of all	k distinct digits, each once: $(k-1)! \times$ digit sum $\times 11\ldots1$

2 AND multiplies, OR adds

The either/or slot is one slot with $5 + 2$ choices.

1 of 4 dals, 1 of 3 sabzis

then 1 of 5 rotis or 1 of 2 rices

$4 \times 3 \times (5 + 2)$

84 **thalis**

Trap: Multiplying the OR too, $4 \times 3 \times 5 \times 2$: 120.

3 Even, no repeats, 0 in the set

Units first, and split on 0.

Digits 0–6, four-digit, even

Ends in 0: $6 \times 5 \times 4 = 120$

Ends in 2, 4, 6: $3 \times (5 \times 5 \times 4) = 300$

$120 + 300 = 420$

Trap: One line, $4 \times 5 \times 5 \times 4$: 400.

4 Two restrictions share a digit

Split on the first digit: an odd first digit uses up an odd ending.

Odd, digits distinct, 3000–6000

First 3 or 5: $2 \times 4 \times 8 \times 7 = 448$

First 4: $1 \times 5 \times 8 \times 7 = 280$

$448 + 280 = 728$

Trap: Five endings every time, $3 \times 5 \times 8 \times 7$: 840.

5 At least one: count the opposite

"No 7 at all" is a clean product.

Four-digit PINs 0000–9999

All: $10^4 = 10000$

No 7: $9^4 = 6561$

At least one 7: 3439

Trap: A 7 fixed in one of 4 places, rest free: 4000 (7777 counted 4 times).

6 Sum of all the numbers

Each digit sits in each place $(k-1)!$ times.

Digits $1, 3, 5, 8$, each once

Each place: every digit $3! = 6$ times

Place sum $= 6 \times 17 = 102$

Total $= 102 \times 1111 = 113322$

Trap: $4!$ instead of $3!$: 453288.

Arrangements

1 Rows, glue, gaps, circles

Row	$n!$; choose r, arrange: ${}^{n}P_{r} = n! \div (n-r)!$
Repeats	Divide by $a!\,b!$ for letters repeated a, b times
Together	Glue into one, then $\times$ the order inside
Apart	Seat the others; drop the group into the gaps
Round table	$(n-1)!$: rotations are the same
Fixed order	k items in a given order: $n! \div k!$



2 Identical letters divide out

Swapping the two Ls changes nothing.

COLLEGE: 7 letters, L twice, E twice

$$\frac{7!}{2! \times 2!} = \frac{5040}{4}$$

1260

Trap: Treating the Ls and Es as different: 5040.

3 Glue the pair

The pair is one token; it can flip.

6 friends, Riya and Kabir together

5 tokens: $5! = 120$

Inside the glue: $\times 2$

240

Trap: Forgetting the flip: 120.

4 Never together: use the gaps

Five boys make six gaps.

5 boys: $5! = 120$

3 girls into 6 gaps: $6 \times 5 \times 4 = 120$

120×120

14400

Trap: All minus "all three together", $8! - 6!\,3!$: 36000.

5 Round a table: fix one

Pin Aman; the rest is a row.

7 seats: $6! = 720$

Aman and Bela glued: $2 \times 5! = 240$

$720 - 240$

480

Trap: Worked as a row, $7! - 2 \times 6!$: 3600.

6 A fixed order: divide

Only one of the three songs' $3!$ orders is allowed.

7 songs, 3 in a fixed order

Gaps between them allowed

$$\frac{7!}{3!} = \frac{5040}{6}$$

840

Trap: Glued into a block, $5!$: 120.

Selections and points

1 Choose: order doesn't matter

A team AB is the team BA. Count down r numbers from n, over $r!$.

Choose	${}^nC_r = \dfrac{n!}{r!\,(n-r)!}$, ${}^nC_r = {}^nC_{n-r}$
Pairs	${}^nC_2 = \dfrac{n(n-1)}{2}$: handshakes, matches, lines
At least k	Cases, or all − the small cases. Never "k first, then anyone"
In or out	Any number of n items: 2^n (the empty choice included)

2 Posts are ordered, members are not

Labelled chairs arrange; a plain bag doesn't.

From 9: president, secretary, 2 members

Posts: $9 \times 8 = 72$

Members: ${}^7C_2 = 21$

$72 \times 21 = 1512$

Trap: Arranging the members too, ${}^9P_4$: 3024.

3 At least 2 women

Take away the 0-woman and 1-woman committees.

5 from 6 men, 4 women

All: ${}^{10}C_5 = 252$

0 women: 6; 1 woman: $4 \times 15 = 60$

$252 - 6 - 60 = 186$

Trap: 2 women first, then any 3, 6×56: 336.

4 Handshakes are pairs

All pairs, minus the forbidden ones.

6 couples: 12 people

All pairs: ${}^{12}C_2 = 66$

No spouse pairs: -6

60

Trap: 12×10, not halved: 120.

5 Must include, can't both

Lock in A; take out the picks holding B and C.

4 songs of 10, A in

With A: ${}^9C_3 = 84$

A, B, C all in: ${}^7C_1 = 7$

$84 - 7 = 77$

Trap: "Not both" read as "neither", ${}^7C_3$: 35.

6 Every item: in or out

Each topping is a switch.

3 crusts, 6 toppings, at least one

Toppings: $2^6 - 1 = 63$

3×63

189

Trap: Keeping the plain pizza, 3×64: 192.

7 Triangles: drop the straight lines

Three posts on one side make no triangle.

6 posts on one side, 4 on the other

${}^{10}C_3 = 120$

Flat: ${}^6C_3 + {}^4C_3 = 24$

96

Trap: Collinear triples kept: 120.

Distributions and paths

1 Shares, groups and paths

Identical, 0 allowed	n stars, $r-1$ bars: ${}^{n+r-1}C_{r-1}$
At least k each	Hand out k each first, then as above
At most m	Take out the shares with someone over m
Different things	r^n; nobody empty: take out, add back
Grid route	a easts, b norths: ${}^{a+b}C_b$



2 Stars and bars

10 stars, 3 bars: 13 places.

10 chocolates, 4 children, 0 allowed

${}^{10+3}C_3$

286

Trap: The at-least-one formula, ${}^{9}C_3$: 84.

3 Minimums first

Give 3 to each, share what's left.

20 notebooks, 4 students, at least 3 each

Left: $20-12=8$

${}^{8+3}C_3$

165

Trap: At-least-one formula, ${}^{19}C_3$: 969.

4 A cap: take out the over-full

Only one child can hold 6 or more.

10 sweets, 3 children, at most 5 each

All: ${}^{12}C_2=66$

One child ≥ 6: ${}^{6}C_2=15$, for each of 3

$66-45=21$

Trap: Taken out for one child only: 51.

5 Everyone gets a gift

Take out the ways that miss a child.

4 different gifts, 3 children

All: $3^4=81$

Miss one: $3\times 2^4=48$; miss two: $+3$

$81-48+3=36$

Trap: One each first, then the 4th: $24\times 3=72$.

6 Pairs have no labels

Divide by the $3!$ orders of the pairs.

6 players into 3 pairs

Labelled: ${}^{6}C_2\times{}^{4}C_2=90$

$90\div 3!$

15

Trap: Pairs as labelled courts: 90.

7 A route is a word of moves

Through a point: leg $\times$ leg.

5 east, 3 north; junction (2, 1) closed

All: ${}^{8}C_3=56$

Through it: ${}^{3}C_1\times{}^{5}C_2=30$

$56-30=26$

Trap: Legs added, $56-13$: 43.