The mixing rule

1 Formula first, then decide

$$C = \frac{V_1C_1 + V_2C_2}{V_1 + V_2}$$

Two or three strengths known	Alligation: the gaps give the ratio, flipped
Only one known	Formula and equations
A fixed amount is added	Not a mixture: plain ratio

20 L of 30% + 30 L of 45%

$$\frac{600 + 1350}{50} = 39\%$$

Trap: The simple average: 37.5%.

2 The alligation web

The net sits between the parents. Gaps, flipped, give the amounts.

10% and 30% to make 25%

Gaps: $30 - 25 = 5, 25 - 10 = 15$

$10\% : 30\% = 5 : 15 = 1 : 3$

Trap: Not flipping: $3 : 1$.

3 One strength known: equations

Two mixes, two unknowns.

$2 : 3$ gives 24%; $3 : 2$ gives 26%

$2a + 3b = 120, 3a + 2b = 130$

$a - b = 10, a + b = 50$

$A = 30\%$

Trap: Averaging the two nets: 25%.

4 The missing parent

Split the known gap in the ratio, then step from the net.

1 part 40% + 2 parts $x \rightarrow 30\%$

Gaps $(30 - x) : (40 - 30) = 1 : 2$

$30 - x = 5 \Rightarrow x = 25\%$

Trap: Gap on the wrong side: 10%.

5 Alligation on prices

Prices work exactly like strengths.

₹40 and ₹60 a kg to cost ₹52

Gaps 8 and 12

Cheap : dear $= 2 : 3$

Trap: Not flipping: $3 : 2$.

Ratio mixtures

1 Milk out of the total

$2:1$ is $\frac{2}{3}$ milk, never 2. One denominator, then alligate.

Flasks tea:milk $2:1$ and $1:1$; want $3:2$

$\frac{2}{3}, \frac{1}{2}, \frac{3}{5} = \frac{20}{30}, \frac{15}{30}, \frac{18}{30}$

Gaps 3 and 2

Flasks $3:2$

Trap: Using $2, 1, 1.5$ as numbers: $1:1$.

2 Volumes that kill fractions

Mixing ratio $\times$ each denominator.

$3:2$ and $4:3$, mixed $2:1$

Take 70 L and 35 L

Lemon $42 + 20 = 62$ of 105

Lemon : water $= 62:43$

Trap: Lemon : total, $62:105$.

3 Equal mix: the midpoint

Same denominator, $1:1$? Average the numerators.

$3:5$ and $7:1 \to \frac{3}{8}, \frac{7}{8}$

Midpoint $\frac{5}{8}$

Milk : water $= 5:3$

Trap: Milk : total, $5:8$.

4 Three ingredients

Alligate on one ingredient. The others follow the same ratio.

A: 60% milk, 10% cocoa; B: 90% milk, no cocoa

75% milk: gaps 15 and $15 \Rightarrow 1:1$

Cocoa $= \frac{10+0}{2} = 5\%$

Trap: Keeping A's cocoa: 10%.

5 Not a mixture

A fixed amount of one liquid is added: just a ratio.

Acid : water $3:2$; add 6 L water $\to 1:1$

$3x = 2x + 6 \Rightarrow x = 6$

Acid $= 18$ **L**

Trap: Answering the total, 30 L.

6 Profit hidden as water

Sells at cost, gains $g\%$: water : milk $= g:100$.

Gain 25%

Water : milk $= 1:4$

Water $= \frac{1}{5} = 20\%$

Trap: Answering 25%.

7 Back and forth

Equal volumes swapped: what each glass gains, the other loses.

300 ml juice, 300 ml water; 60 ml over, 60 ml back

Back pour: $60 \times \frac{60}{360} = 10$ juice, 50 water

Water in the juice glass $= 50$ **ml**

Trap: The full 60 ml.

Removal and replacement

1 The left-over factor

Follow only what goes out and never comes back.

$$\text{Left} = \text{Start} \times \left(1 - \frac{\text{out}}{V}\right)^{n}$$

64 L milk; 16 L out, water in, 3 times

$64 \times \left(\frac{3}{4}\right)^{3}$

$= 27$ **L**

Trap: Subtracting 16 each time: 16 L.

2 Starting impure

Same factors on the starting share.

50 L of 80%; 10 L out, water in, twice

$80 \times \left(\frac{4}{5}\right)^{2}$

$= 51.2\%$

Trap: Dropping 10 points a time: 60%.

3 Work backwards with k^n

k is the fraction **left**, not taken.

Pure milk; 12 L out, water in, 3 times; 8 : 19

Milk $= \frac{8}{27} = k^3 \Rightarrow k = \frac{2}{3}$

$\frac{1}{3}$ drawn $= 12$ L

Capacity $= 36$ **L**

Trap: Taking k as the fraction drawn: 18 L.

4 Refilled with milk? Stop

The chain only works while the refill is water.

100 L milk: 20 L out, $\frac{1}{4}$ out (water each)

Milk $= 100 \times \frac{4}{5} \times \frac{3}{4} = 60$

40% out: milk 36; add 40 milk

Milk $= 76$ **L**

Trap: Multiplying through: 36 L.

5 Refilled with a solution

It is just a mixture of what stays and what comes in.

20 L of 80%; a quarter replaced with 40%

15 L at 80% + 5 L at 40%

$= 70\%$

Trap: Treating the refill as water: 60%.

6 Fewest replacements

Step down the powers until you pass the line.

50 L milk; 10 L out each time; below 30%?

$0.8^5 = 0.328, 0.8^6 = 0.262$

6 **times**

Trap: A straight 20% drop: 4 times.

7 Before you answer

Two strengths known? Alligate. One? Equations. · Ratios: milk out of the total first. · Asked milk : water or milk : total? · Refill not water? Stop the factor chain.