Simple interest

1 Interest is a share of the principal

One year	$I = r\%$ of P, the same every year
T **years**	$SI = rT\%$ of P
Amount	$P \times \left(1 + \frac{rT}{100}\right)$

40% in 5 years, so 8% a year

Three times itself means 200% interest

$200 \div 8 = 25$ **years**

Trap: Counting the principal as interest: $300 \div 8 = 37.5$ years.

2 SI is an AP

Each year adds the same I. Two amounts give I, then step back.

₹$9{,}200$ after 3 years, ₹$10{,}800$ after 7

$I = \frac{1{,}600}{4} =$ ₹400

$P = 9{,}200 - 3 \times 400 =$ ₹$8{,}000$

Trap: Taking off the whole ₹$1{,}600$: ₹$7{,}600$.

3 Two accounts, one total

It is a mixture. Find the net rate, then alligate.

₹$50{,}000$ at 6% and 10%; interest ₹$4{,}200$

Net 8.4%; gaps 1.6 and 2.4

$6\% : 10\% = 2 : 3$

At 6%**:** ₹$20{,}000$

Trap: Not flipping the gaps: ₹$30{,}000$.

4 Same interest from each account

Equal interest means money goes as $\frac{1}{r}$.

₹$66{,}000$ at 5%, 10%, 15%

$\frac{1}{5} : \frac{1}{10} : \frac{1}{15} = 6 : 3 : 2$

At 5%**:** $6 \times 6{,}000 =$ ₹$36{,}000$

Trap: Sharing as $1 : 2 : 3$: ₹$11{,}000$.

5 Reinvest: multiply, don't add

The amount from one bank is the next bank's principal.

₹$20{,}000$: 10% SI for 3 years, then 5% SI for 4

$20{,}000 \times 1.3 \times 1.2$

$=$ ₹$31{,}200$

Trap: Adding $30\% + 20\%$: ₹$30{,}000$.

Compound interest

1 One factor, three shortcuts

$$A = P\left(1 + \frac{r}{100}\right)^n$$

Write the rate as a fraction first: $5\% = \frac{1}{20}$, $8\% = \frac{2}{25}$, $10\% = \frac{1}{10}$, $12.5\% = \frac{1}{8}$, $20\% = \frac{1}{5}$, $25\% = \frac{1}{4}$.

P given	$2I + I'$ for 2 years, $3I + 3I' + I''$ for 3; each is $r\%$ of the one before
A given	Ratio and assumption: rate $\frac{1}{k}$ means $P : A = k^n : (k+1)^n$
Only r	Successive percentage: 2-year CI is $2r + \frac{r^2}{100}$ percent

₹12,000 at 15% for 2 years

$I = 1{,}800$, $I' = 15\%$ of $1{,}800 = 270$

$CI = 3{,}600 + 270 = ₹3{,}870$

Trap: Stopping at $2I$, the simple interest: ₹3,600.

2 Ratio and assumption

Amount given? Assume P is k^n.

20% CI; ₹7,200 after 2 years

$5 \to 6$, so $25 \to 36$; one part $= 200$

$P = 25 \times 200 = ₹5{,}000$

Trap: Taking 20% off twice: ₹4,608.

3 SI and CI side by side

Each part earns its own net percentage.

₹60,000: $\frac{2}{3}$ at 10% SI, rest at 20% CI, 2 years

SI: 20% of $40{,}000 = 8{,}000$

CI: $20 + 20 + 4 = 44\%$ of $20{,}000 = 8{,}800$

Total ₹16,800

Trap: CI part as simple: ₹16,000.

4 Three years: $3I + 3I' + I''$

Three of I, three of I', one of I''.

₹30,000 at 10% for 3 years

$I = 3{,}000$, $I' = 300$, $I'' = 30$

$9{,}000 + 900 + 30 = ₹9{,}930$

Trap: Only $3I$: ₹9,000.

5 Fraction cube backwards

Three years from the amount: cube the fraction.

25% CI; ₹25,000 after 3 years

$4 \to 5$, so $64 \to 125$; one part $= 200$

$P = 64 \times 200 = ₹12{,}800$

Trap: Taking 25% off three times: ₹10,547.

Chains and the yearly ladder

1 CI is a GP

The interest earned in one year grows by the same factor as the amount.

$$I_n = (r\% \text{ of } P) \times \left(1 + \frac{r}{100}\right)^{n-1}$$

10% CI; interest in year 2 is ₹$2{,}200$

Year 4 is two steps on: $2{,}200 \times 1.1 \times 1.1$

$= ₹2{,}662$

Trap: Adding ₹220 a year, as in SI: ₹$2{,}640$.

2 Two year-interests give the sum

Divide to get the factor, step back to year 1.

Year 3: ₹$7{,}200$; year 5: ₹$10{,}368$

$\frac{10{,}368}{7{,}200} = 1.44 = 1.2^2$, so 20%

Year 1: $\frac{7{,}200}{1.44} = 5{,}000 = 20\%$ of P

$P = ₹25{,}000$

Trap: Year 3 as year 1: ₹$36{,}000$.

3 CI then SI

A chain of banks: one factor per bank.

₹$25{,}000$: 20% CI for 2 years, then 10% SI for 3

$25{,}000 \times 1.44 = 36{,}000$

$36{,}000 \times 1.3 = ₹46{,}800$

Trap: Compounding the SI leg: ₹$47{,}916$.

4 Every six months

Half the rate, twice the periods. Quarterly: a quarter, four times.

₹$50{,}000$ at 20% a year, half-yearly, 2 years

10% four times: $50{,}000 \times 1.1^4 = 73{,}205$

Interest ₹$23{,}205$

Trap: Compounding yearly: ₹$22{,}000$.

5 Rate changes mid-way

Treat it as moving the money to a new bank.

5% for 2 years, then 4% for 3

$A = P \times 1.05^2 \times 1.04^3$

Trap: Averaging the rates.

The SI–CI gap and instalments

1 The gap is interest on interest

2 years	$CI - SI = I' = P\left(\frac{r}{100}\right)^2$
3 years	$CI - SI = 3I' + I'' = I'\left(3 + \frac{r}{100}\right)$

Find I', then go up to P by $\frac{100}{r}$ twice.

2 The two-year gap

5%; 2-year gap ₹$75 = I'$

$I = 75 \times 20 = 1{,}500$

$P = 1{,}500 \times 20 = ₹30{,}000$

Trap: Going up once: ₹$1{,}500$.

3 The three-year gap

20%; 3-year gap ₹$3{,}200$

$I' = \frac{3{,}200}{3.2} = 1{,}000$; $I = 5{,}000$

$P = 5{,}000 \times 5 = ₹25{,}000$

Trap: Using the 2-year rule: ₹$80{,}000$.

4 Pay it down year by year

Grow a year, subtract the payment, repeat until zero.

₹$16{,}000$ at 20%; pay x, then $2x$

Year 1: $19{,}200 - x$

Year 2: $1.2(19{,}200 - x) = 2x$

$23{,}040 = 3.2x$, **so** $x = ₹7{,}200$

Trap: Growing the whole loan, then splitting $1:2$: ₹$7{,}680$.

5 Deposits at the start of the year

Each deposit earns until the end. Start-of-year deposits get one year more.

₹$1{,}000$ at the start of each year, 3 years, 10%

$1{,}331 + 1{,}210 + 1{,}100$

$= ₹3{,}641$

Trap: As end-of-year deposits: ₹$3{,}310$.

6 Before you answer

SI: same interest every year, add. CI: same factor every year, multiply. · Money in several places? Net rate, then alligate. · Half-yearly? Half the rate, twice the periods. · 2-year or 3-year gap? Pick the right rule. · Loans: follow the balance, not a formula.