Eight solids, one table

1 Every formula is faces you can count

Curved surface is the part you could roll on a table. Total adds the flat faces. Paint means surface, fill means volume.

	Solid	Curved or side surface	Total surface	Volume
	Cuboid l, b, h	$2h(l+b)$	$2(lb+bh+hl)$	lbh
	Cube a	$4a^2$	$6a^2$	a^3
	Cylinder r, h	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
	Cone r, h, l	πrl, with $l=\sqrt{h^2+r^2}$	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
	Sphere r	$4\pi r^2$ (four circles)	$4\pi r^2$	$\frac{4}{3}\pi r^3$
	Hemisphere r	$2\pi r^2$	$3\pi r^2$ (with the lid)	$\frac{2}{3}\pi r^3$
	Frustum R, r, h	$\pi(R+r)l$, with $l=\sqrt{h^2+(R-r)^2}$	add $\pi R^2+\pi r^2$	$\frac{1}{3}\pi h(R^2+Rr+r^2)$
	Pyramid, base B	$\frac{1}{2}\times$ perimeter $\times$ slant	add B	$\frac{1}{3}Bh$

2 Volume = base area × height

Works whenever the cross-section is the same all the way along: cuboid, cylinder, any prism.

Pool 20×8 m, depth 1 m to 3 m

Side wall: trapezium $\frac{1+3}{2} \times 20 = 40$ m²

Water $= 40 \times 8 = 320$ **m³**

Trap: 3 m deep everywhere: 480 m³.

3 A cone is a third of its cylinder

Same base, same height: cone $= \frac{1}{3}$ cylinder.

Pouring keeps the volume.

Cone $r=6, h=15$ into a can of $r=3$

$\frac{1}{3}\pi \cdot 36 \cdot 15 = 180\pi$

$\pi \cdot 9 \cdot H = 180\pi$

$H = 20$ **cm**

Trap: Forgetting the $\frac{1}{3}$: 60 cm.

4 Radius counts twice

$V \propto r^2 h$. Multiply the factors, never add the percentages.

r up 20%, h down 20%

$1.2^2 \times 0.8 = 1.44 \times 0.8 = 1.152$

Volume $+15.2\%$

Trap: $+20-20=0\%$.

5 Hollow pipe: a ring times length

Ring $= \pi(R^2-r^2) = \pi(R+r)(R-r)$.

70 cm long, diameters 10 and 8, $\pi = \frac{22}{7}$

Ring $= \pi(5+4)(5-4) = 9\pi$

Iron $= \frac{22}{7} \times 9 \times 70 = 1980$ **cm³**

Trap: $\pi(R-r)^2 \times 70$: 220 cm³.

Nets and faces

1 Cut it open: the curved face is a rectangle

Slit a cylinder down its side and flatten it. The sheet is the circumference $2\pi r$ long and h tall, so its area is $2\pi rh$.

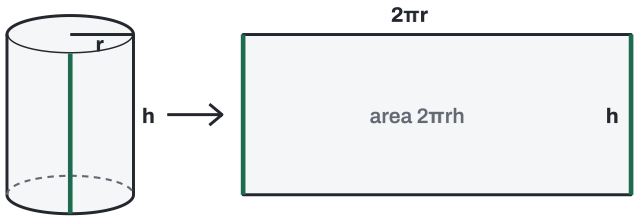
Sheet 44×20, rolled both ways

$V = \pi r^2 h = \dfrac{C^2 h}{4\pi}$, and $C \times h = 880$ is fixed

So $V \propto C$

44 round: $r = 7$, $V = 3080$; 20 round: $V = 1400$

Ratio $= 44 : 20 = 11 : 5$



Trap: "Same sheet, same volume": $1 : 1$.

2 Fold the card: every side loses two cuts

Cut x from each corner: the base shrinks by $2x$ both ways and the height is x.

Card 30×20, corners 5 cm

Base $(30 - 10) \times (20 - 10) = 20 \times 10$

Tray $= 20 \times 10 \times 5 = 1000$ **cm³**

Trap: $30 \times 20 \times 5 = 3000$ cm³.

3 Three faces give the box

$(lb)(bh)(hl) = (lbh)^2$, so $V = \sqrt{\text{product of the three faces}}$. Each edge is V over the face it is not on.

Faces $48, 32, 24$

$V = \sqrt{48 \times 32 \times 24} = 192$

Edges $\dfrac{192}{24} = 8$, $\dfrac{192}{32} = 6$, $\dfrac{192}{48} = 4$

All 12 edges $= 4 \times 18 = 72$ **cm**

Trap: Stopping at $8 + 6 + 4 = 18$.

4 A sector curls into a cone

The sector's radius becomes the slant l. Its arc becomes the base circumference.

Radius 10, angle 216°

Arc $= \dfrac{216}{360} \times 2\pi \times 10 = 12\pi = 2\pi r$

$r = 6$, $l = 10$, so $h = \sqrt{100 - 36} = 8$

$V = \dfrac{1}{3}\pi \times 36 \times 8 = 96\pi$ **cm³**

Trap: Slant 10 as the height: 120π.

5 Glued faces vanish

For a joined solid, list only the faces you can touch. The joint disc is inside.

Silo: cylinder $r = 3$, $h = 10$, dome on top, floor too

Wall $2\pi \times 3 \times 10 = 60\pi$

Dome $2\pi \times 9 = 18\pi$; floor 9π

Coated $= 87\pi$ **m²**

Trap: Two full TSAs, $78\pi + 27\pi = 105\pi$.

6 Slant height is not height

Half the base, height and slant form a right triangle.

Square pyramid, base 16, face slant 10

$h = \sqrt{10^2 - 8^2} = 6$

$V = \dfrac{1}{3} \times 256 \times 6 = 512$ **m³**

Trap: Slant as the height: $853\frac{1}{3}$ m³.

Melt, cut, sink

1 Melting keeps the volume, not the surface

Melt, recast, pour or mould: set old volume = new volume and let π cancel. The surface is free to change.

Melt into n pieces	$n = \dfrac{\text{big volume}}{\text{one small volume}}$
Same shape, $\frac{1}{k}$ the size	$n = k^3$ pieces, total surface $\times k$
Part of the metal wasted	Scale the volume first, then count

Rod $r = 4$, length 16 into balls of diameter 4

Rod $= \pi \times 16 \times 16 = 256\pi$

Ball $(r = 2) = \dfrac{4}{3}\pi \times 8 = \dfrac{32}{3}\pi$

$n = 256 \times \dfrac{3}{32} = 24$ **balls**

Trap: Diameter 4 used as the radius: 3 balls.

2 Cuts expose new faces

Cut a cube into k^3 equal cubes: each piece has $\dfrac{1}{k^2}$ the area, so the total is k times.

12 cm cube into 27 cubes of 4 cm

Before: $6 \times 144 = 864$

After: $27 \times 6 \times 16 = 2592$

3 **times** $(k = 3)$

Trap: 27 pieces, 27 times.

3 Painted block: peel each side

Painted $l \times b \times h$, cut into unit cubes. The unpainted core loses one layer from each side.

$5 \times 4 \times 3$ block

Core $(5-2)(4-2)(3-2)$

$3 \times 2 \times 1 = 6$ **unpainted**

Trap: $(5-1)(4-1)(3-1) = 24$.

4 The solid that sticks out

A block taller than the water doesn't add its whole volume. The water just sits on a smaller floor.

Tank 50×40, water 12 deep, cube 20

Water $= 2000 \times 12 = 24000$

Floor left $= 2000 - 400 = 1600$

Depth $= \dfrac{24000}{1600} = 15 < 20$: it sticks out

New depth $= 15$ **cm**

Trap: Adding $\dfrac{8000}{2000} = 4$: 16 cm.

5 Painted $n \times n \times n$ cube, cut into n^3

3 **faces**	8, the corners
2 **faces**	$12(n-2)$, the edges
1 **face**	$6(n-2)^2$, mid-faces
None	$(n-2)^3$, the core

$n = 5$: $8 + 36 + 54 + 27$

$= 125 = 5^3$**, all counted once**

Scale, cones and boxes

1 Lengths k, areas k^2, volumes k^3

For similar solids, every length scales by the same k. Surface, paint and cloth scale by k^2. Volume, weight and capacity scale by k^3.

Model tank: lengths $\frac{1}{20}$, holds 2 L

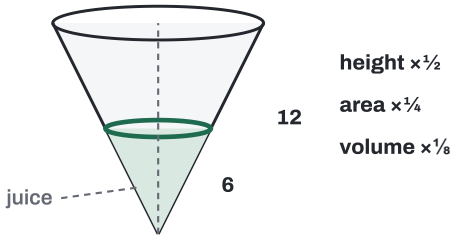
Real $= 2 \times 20^3$

16000 **litres**

Cone glass 12 cm tall, juice 6 cm deep

Juice is a cone $\frac{1}{2}$ the size: $\left(\frac{1}{2}\right)^3$

$\frac{1}{8}$ **full**



Trap: k^2 for a volume: 800 L; half the height as half full.

2 A frustum is a cone minus a cone

Extend the walls to the tip. The radii ratio gives the full height.

Bucket: $r = 10$, $R = 20$, depth 30

$\frac{10}{20} = \frac{1}{2}$: full cone 60 tall, tip 30

$\frac{1}{3}\pi(400 \times 60 - 100 \times 30)$

$= \frac{1}{3}\pi \times 21000 = 7000\pi$ **cm³**

Check: $\frac{1}{3}\pi \times 30 \times (400 + 200 + 100) = 7000\pi$

Trap: Cylinder on radius 15: 6750π.

3 The space diagonal

Longest rod in a box $= \sqrt{l^2 + b^2 + h^2}$: the floor diagonal, then up the corner.

Hall $12 \times 9 \times 8$

Floor: $\sqrt{144 + 81} = 15$

Up the corner: $\sqrt{15^2 + 8^2}$

$= 17$ **m**

Trap: The floor diagonal: 15 m.

4 $(l + b + h)^2$ holds the surface

$$(l + b + h)^2 = \underbrace{l^2 + b^2 + h^2}_{d^2} + \underbrace{2(lb + bh + hl)}_{\text{TSA}}$$

Diagonal 13, all edges 76

$l + b + h = \frac{76}{4} = 19$

TSA $= 19^2 - 13^2 = 361 - 169$

$= 192$ **cm²**

Trap: Halving it: 96.

5 Snug fits: a ball is $\frac{2}{3}$ of its cylinder

A ball in a cylinder that just holds it: cone : ball : cylinder $= 1 : 2 : 3$.

3 balls of $r = 6$ in a tube

Tube $= \pi \times 36 \times 36 = 1296\pi$

Balls $= 3 \times \frac{4}{3}\pi \times 216 = 864\pi$

Empty $= 432\pi$ **cm³, a third**

Trap: $4\pi r^2$ as the volume: 864π.

6 Before you answer

Curved or total? · Radius or diameter? · Slant or height? · Melting keeps volume, cutting grows surface. · Does the block stick out of the water? · Scale: k, k^2, k^3.