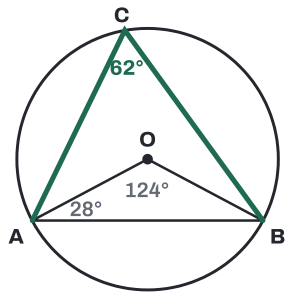
Angles in a circle

1 Draw the radii first

Every triangle with two radii is isosceles. A chord's angle sits at a third point, never at its own ends.

Centre	Twice the angle on the circle, same arc
Diameter	Makes $90°$ on the circle
Same arc	Equal angles on the same side
Cyclic quad	Opposite angles add to $180°$; outside angle $=$ inside opposite
Equal chords	Equal angles, at the centre and on the circle



2 The centre angle is double

Base angles of $\triangle OAB$ first, then halve.

C on the major arc, $\angle OAB = 28°$

$OA = OB$, so $\angle OBA = 28°$

$\angle AOB = 180° - 2 \times 28° = 124°$

$\angle ACB = 124° \div 2 = 62°$

Trap: Stopping at the centre angle: $124°$.

3 The angle in a semicircle

The other two angles share only $90°$.

AB a diameter, $\angle CAB = 3\angle CBA$

$\angle ACB = 90°$

$x + 3x = 90°$

$\angle CBA = 22.5°$

Trap: $x + 3x = 180°$: $45°$.

4 Same arc, same angle

The crossing angle is outside $\triangle ABP$.

AC, BD cross at P; $\angle BAC = 35°$, $\angle BPC = 80°$

$80° = 35° + \angle ABD$, so $\angle ABD = 45°$

$\angle ACD = \angle ABD = 45°$ **(arc AD)**

Trap: $80°$ inside the triangle: $65°$.

5 The cyclic quadrilateral's outside angle

Outside angle $=$ the opposite corner's angle.

AB extended to E; $\angle CBE = 76°$, $\angle ADB = 28°$

$\angle ADC = \angle CBE = 76°$

$\angle BDC = 76° - 28°$

$\angle BDC = 48°$

Trap: $\angle ADC = 180° - 76°$: $76°$.

6 Equal chords, equal angles

Same angle on the circle, twice it at O.

$AB = BC = CD$, $\angle AEB = 22°$

Each chord: $22°$ at E, $44°$ at O

$\angle AOD = 3 \times 44°$

$\angle AOD = 132°$

Trap: Forgetting the centre doubles: $66°$.

7 Two tangents and the centre

Radius $\perp$ tangent, so $\angle AOB = 180° - \angle P$.

Tangents from P touch at A, B; $\angle APB = 52°$

$\angle AOB = 360° - 90° - 90° - 52° = 128°$

C on the major arc: half of it

$\angle ACB = 64°$

Trap: Stopping at $\angle AOB$: $128°$.

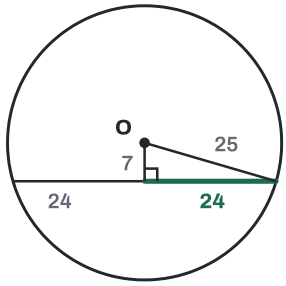
Chords and tangents

1 A chord is two right triangles

The perpendicular from the centre cuts a chord in half. Half chord, distance and radius make a right triangle, so think in triplets: $7, 24, 25$ and $15, 20, 25$.

Chord	$\left(\frac{L}{2}\right)^2 + d^2 = r^2$
Two parallel chords	Same side: $d_1 - d_2$ apart. Opposite sides: $d_1 + d_2$
Crossing chords	$AP \times PB = CP \times PD$
Tangent	Meets the radius at $90°$; two from a point are equal; $OP^2 = r^2 + L^2$

Radius 25, chord 7 from the centre
Half chord $= \sqrt{25^2 - 7^2}$
$= \sqrt{576} = 24$
Chord $= 2 \times 24 = 48$ **cm**



Trap: Stopping at the half chord: 24 cm.

2 Two parallel chords

Find each chord's distance from the centre, then subtract (same side) or add (opposite sides).

Radius 25; chords 48 and 40, same side
Chord 48: $d = \sqrt{625 - 576} = 7$
Chord 40: $d = \sqrt{625 - 400} = 15$
Apart: $15 - 7 = 8$ **cm**

Trap: Adding, as if on opposite sides: 22 cm.

3 Crossing chords

The two pieces of one chord multiply to the same number as the pieces of the other.

$AP = 6, PB = 8, CD = 16, CP < PD$
$CP \times PD = 6 \times 8 = 48$
$CP + PD = 16$: the pair 4 and 12
$CP = 4$ **cm**

Trap: $48 \div 16$: 3 cm.

4 The tangent kite

OP splits the kite into two equal right triangles, so its area is $r \times L$.

$r = 10, OP = 26$
$PA = \sqrt{26^2 - 10^2} = 24$
Kite $= 2 \times \frac{1}{2} \times 10 \times 24$
$PAOB = 240$ **cm²**

Trap: One triangle only: 120 cm².

5 A chord touching a smaller circle

Two circles share a centre and a chord of the big one touches the small one. Then $R^2 - r^2 = \left(\frac{L}{2}\right)^2$, so the ring needs no radius.

Chord of the big circle $= 24$, touching the small one
$R^2 - r^2 = 12^2 = 144$
Ring $= \pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right)$
Ring $= 144\pi$ **cm²**

Trap: The whole chord squared: 576π.

Arcs, sectors, segments

1 A sector is a share of the circle

θ out of 360° is that share of the area and of the rim. Segment = sector − triangle; CAT keeps to 60° (equilateral, chord $= r$) and 90° (chord $= r\sqrt{2}$).

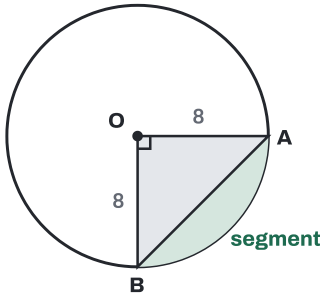
Circle	Area πr^2, rim $2\pi r$
Sector, angle θ	Area $\frac{\theta}{360}\pi r^2$, arc $\frac{\theta}{360}2\pi r$, perimeter arc $+ 2r$
Semicircle	Perimeter $\pi r + 2r$: the diameter counts
Segment	60°: $\frac{\pi r^2}{6} - \frac{\sqrt{3}}{4}r^2$ $\quad 90^\circ$: $\frac{\pi r^2}{4} - \frac{r^2}{2}$

Radius 8, chord AB makes 90° at the centre

Sector $= \frac{90}{360} \times \pi \times 64 = 16\pi$

Triangle $= \frac{1}{2} \times 8 \times 8 = 32$

Minor segment $= (16\pi - 32)$ **cm²**



Trap: Stopping at the sector: 16π.

2 Sector perimeter

The perimeter is the arc and the two straight edges.

Radius 15, angle 48°

Arc $= \frac{48}{360} \times 2\pi \times 15 = 4\pi$

Perimeter $= 15 + 15 + 4\pi$

$(30 + 4\pi)$ **cm**

Trap: The arc only: 4π.

3 The major segment

The big piece is the whole circle minus the small segment, so the triangle is added back.

Radius 12, chord 12: equilateral, 60°

Minor $= \frac{144\pi}{6} - \frac{\sqrt{3}}{4} \times 144 = 24\pi - 36\sqrt{3}$

Major $= 144\pi - (24\pi - 36\sqrt{3})$

$(120\pi + 36\sqrt{3})$ **cm²**

Trap: Major sector minus the triangle: $120\pi - 36\sqrt{3}$.

4 Two circles through each other's centres

The two centres and a crossing point are all r apart, so the angle at each centre is $2 \times 60^\circ = 120^\circ$. The common region is two 120° segments.

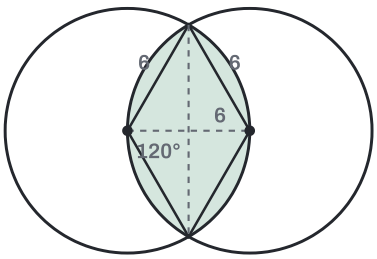
Two circles of radius 6, each through the other's centre

Sector $= \frac{120}{360} \times 36\pi = 12\pi$

Its triangle: base $6\sqrt{3}$, height 3, area $9\sqrt{3}$

One segment $= 12\pi - 9\sqrt{3}$

Common region $= (24\pi - 18\sqrt{3})$ **cm²**



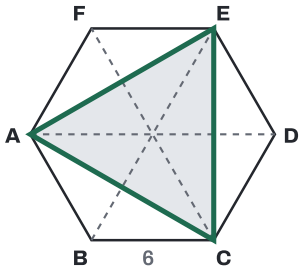
Trap: 60° segments: $12\pi - 18\sqrt{3}$.

Polygons and circles

1 Walk round the outside

Walking round a convex polygon you turn 360° in all. Count turns, not insides.

Exterior	Add to 360°; each is $180^\circ -$ interior
Interior	Add to $(n-2) \times 180^\circ$
Diagonals	$\frac{n(n-3)}{2}$: $n-3$ per corner, each counted twice
Hexagon	Six equilateral triangles: area $6 \times \frac{\sqrt{3}}{4}s^2$; radii s and $\frac{\sqrt{3}}{2}s$



2 Exterior angles add to 360

One odd angle: count its turn separately.

One angle 100°, the others 160°

Turns: 80° once, 20° for the rest

$80 + 20(n-1) = 360$, so $n - 1 = 14$

$n = 15$ **sides**

Trap: Every angle as 160°: 18.

3 Counting diagonals

Set $\frac{n(n-3)}{2}$ to what is given.

Diagonals $= 4\times$ sides

$\frac{n(n-3)}{2} = 4n$

$n - 3 = 8$

$n = 11$ **sides**

Trap: Dropping the half: 7.

4 The hexagon's big triangle

ACE is exactly half the hexagon.

Side 6

Hexagon $= 6 \times \frac{\sqrt{3}}{4} \times 36 = 54\sqrt{3}$

$\triangle ACE = \frac{1}{2} \times 54\sqrt{3}$

$27\sqrt{3}$ **cm²**

Trap: AC taken as the long diagonal 12: $36\sqrt{3}$.

5 An octagon from a square

Each corner: right isosceles, long side 4.

Octagon side 4

Corner leg $= \frac{4}{\sqrt{2}} = 2\sqrt{2}$

Square side $= 2\sqrt{2} + 4 + 2\sqrt{2}$

$(4 + 4\sqrt{2})$ **cm**

Trap: One leg added, not two: $4 + 2\sqrt{2}$.

6 The hexagon's two circles

Radii: a small triangle's height and side.

Inner circle $= 27\pi$

$r^2 : R^2 = \left(\frac{\sqrt{3}}{2}s\right)^2 : s^2 = 3 : 4$

Outer $= 27\pi \times \frac{4}{3}$

36π **cm²**

Trap: A square's ratio, $\times 2$: 54π.

7 Two circles on a line

Touch points are $2\sqrt{r_1 r_2}$ apart.

Radii 9 and 4, touching, on one line

Centres 13 apart, heights differ by 5

$\sqrt{13^2 - 5^2} = \sqrt{144}$

12 **cm** $= 2\sqrt{9 \times 4}$

Trap: The sum of the radii: 13.