Sum first

1 Write the sum, then move it

$$\text{Sum} = \text{Count} \times \text{Average}$$

One joins	Newcomer = new average + $n\times$ rise
One leaves	Leaver = old average + $(n-1)\times$ fall
One replaced	New − old = $n\times$ change

$72, 78, 81, 69, 85$: sixth score lifts the average by 2

Deviations from 75: $-3, +3, +6, -6, +10 \Rightarrow$ average 77

New average 79; the five each need 2 more

Sixth $= 79 + 5 \times 2 = 89$

Trap: Answering the new average, 79.

2 Someone leaves

The ones who stay share the fall. There are $n - 1$ of them.

15 runners average 42 min

Slowest leaves; average falls by 1.5

$14 \times 1.5 = 21$ min handed back

Slowest $= 42 + 21 = 63$ **min**

Trap: Using 15×1.5: 64.5 min.

3 Replace one, the count stays

The whole change sits on one seat.

Crew of 10; a 92 kg rower is replaced

Average falls by 1.5 kg

Weight lost $= 10 \times 1.5 = 15$ kg

New rower $= 92 - 15 = 77$ **kg**

Trap: Taking off 1.5 only: 90.5 kg.

4 The average moves by a whole number

Old average $+ \dfrac{\text{extra}}{\text{new count}}$. The new count must divide the extra.

Average age 24; $31, 35$ retire; $20, 22, 25$ join

Extra $= 67 - 66 - 24 = -23$

Fall $= \dfrac{23}{n+1}$, whole $\Rightarrow n + 1 = 23$

Players at first $= 22$

Trap: Answering the new count, 23.

5 Track the total change

Total change = count $\times$ change in average. Then share it out.

40 students, $62 \to 64$: $+80$ marks

Top 10 lose 4 each: -40

Other 30 must add 120

$x = \dfrac{120}{30} = 4$

Trap: Giving each the change in average, 2.

Groups and runs

1 One move, both averages rise

Below P's average and above Q's? Both go up.

$$w = a - (p-1)d = b + (q+1)d$$

P: 9 at 62 kg. Q: 12 at 41 kg

$62 - 8d = 41 + 13d \Rightarrow d = 1$

Weight $= 62 - 8 = 54$ **kg**

Trap: Using 9 and 12: 53 kg.

2 Leave-one-out averages

Each number is left out once. The averages add up to the total.

Others average $21, 23, 24, 26, 26$

Total $= 21 + 23 + 24 + 26 + 26 = 120$

Number $= 120 - 4\times$ its average

Largest $= 120 - 4 \times 21 = 36$

Trap: Using the largest average: 16, the smallest.

3 The average is the middle

In an AP, average $=$ middle term $= \dfrac{\text{first} + \text{last}}{2}$.

9 consecutive odd numbers, sum 351

Middle $= \dfrac{351}{9} = 39$

Four steps of 2 up

Largest $= 39 + 8 = 47$

Trap: Steps of 1: 43.

4 Half of a run

The two middles are $\dfrac{n-k}{2}$ places apart. Times the step.

10 consecutive even numbers average 47

Smallest 5: middle moves 2.5 places

$2.5 \times 2 = 5$

Average $= 47 - 5 = 42$

Trap: Forgetting the step of 2: 44.5.

5 The erased number

The average of 1 to n is $\dfrac{n+1}{2}$. One erased barely moves it, and the denominator must divide $n-1$.

1 to n, one erased, the rest average $20.6 = \dfrac{103}{5}$

$n \approx 40$, and 5 divides $n-1 \Rightarrow n = 41$

Sum 1 to $41 = 861$; the rest $= 40 \times 20.6 = 824$

Erased $= 861 - 824 = 37$

Trap: Answering n, 41.

Time, cricket, extremes

1 Everyone gets older

After t years, the sum grows by $n \times t$.

5 years ago: 5 people averaged 26

Today the five sum $5 \times 31 = 155$

With the baby: $6 \times 26 = 156$

Baby $= 156 - 155 = 1$ **year**

Trap: Forgetting the years: a baby of 26.

2 Same average after years

The swap took away $n \times t$ years.

8 members, average 45 six years ago and today

Ageing added $8 \times 6 = 48$ years

New member is 48 years younger

Trap: Answering 6.

3 Not out doesn't count

$$\text{Batting average} = \frac{\text{Runs}}{\text{Dismissals}}$$

40 over 15 outs: 600 runs

Then 72, 29 not out, 64: $+165$ runs, $+2$ outs

$\frac{765}{17} = 45$

Trap: Dividing by 18 innings: 42.5.

4 Median is not the mean

Push the small ones down, then climb 1 apart from the median.

9 distinct positive integers, average 30: sum 270

$1 + 2 + 3 + 4 + m + (m+1) + \cdots + (m+4)$

$20 + 5m = 270$

Median $m = 50$

Trap: Median $=$ mean: 30.

5 How many can sit below a line

Low ones just under the line. The rest at the cap.

8 scores out of 100, average 60: sum 480

k at 39, the rest at 100: $800 - 61k \geq 480$

$k \leq \frac{320}{61} \approx 5.2$

At most 5 below 40

Trap: Low ones at 0: only 3.

6 Before you answer

Write the sum first. · Who shares the change: n, $n-1$ or $n+1$? · Old count or new count? · Time passed? Add $n \times t$. · Two groups of different sizes? That's alligation (Mixtures sheet).